Park Hill Thorns Federation

Routes through calculations

November 2023

Overview

These end points are based on DfE Guidance “Teaching Mathematics in Primary Schools” (2020)

	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combine numbers within 10 (aggregation)	Add numbers within 10	Add two numbers, bridging 10. Add two-digit numbers	Add three-digit numbers using columnar methods and exchange	Add four-digit numbers using columnar methods and exchange	Add decimal numbers using columnar methods and exchange	
Subtraction	Understand the composition of numbers to 10	Partition numbers within 10	Subtract two numbers, bridging 10. Subtract two-digit numbers.	Subtract three-digit numbers using columnar methods and exchange	Subtract four-digit numbers using columnar methods and exchange	Subtract decimal numbers using columnar methods and exchange	
Multiplication			Calculate products within the 2, 5 and 10 times tables by repeated addition	Solve multiplication problems within known times tables	Understand multiplication principles such as the distributive law	Multiply any number (including decimals) by a single-digit using short multiplication.	Multiply any whole number by two-digit number using long multiplication
Division			Understand division as finding how many equal groups are in a number.	Understand division as grouping and as sharing.	Find and interpret remainders in division problems.	Divide any whole number by a single digit using short division (with remainders)	Divide any whole number by a two-digit number using short or long division, as appropriate.

Fluency in number facts

Number facts to 20 are learned and practiced in Years 1-3. By the end of Year 3, a child should have automatic recall of all of the following facts, as well as related subtraction facts:

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

Adding 1

Adding 2

Bonds to 10

Adding 0

Adding 10

Doubles

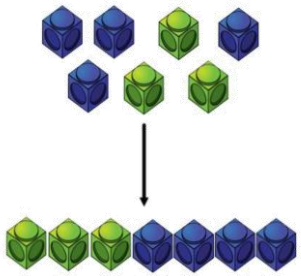
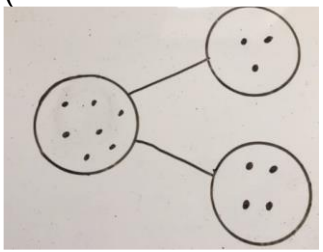
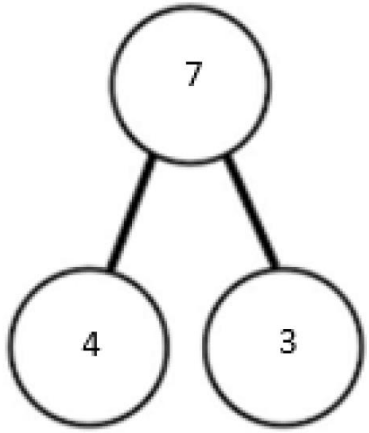
Bridging/ compensating

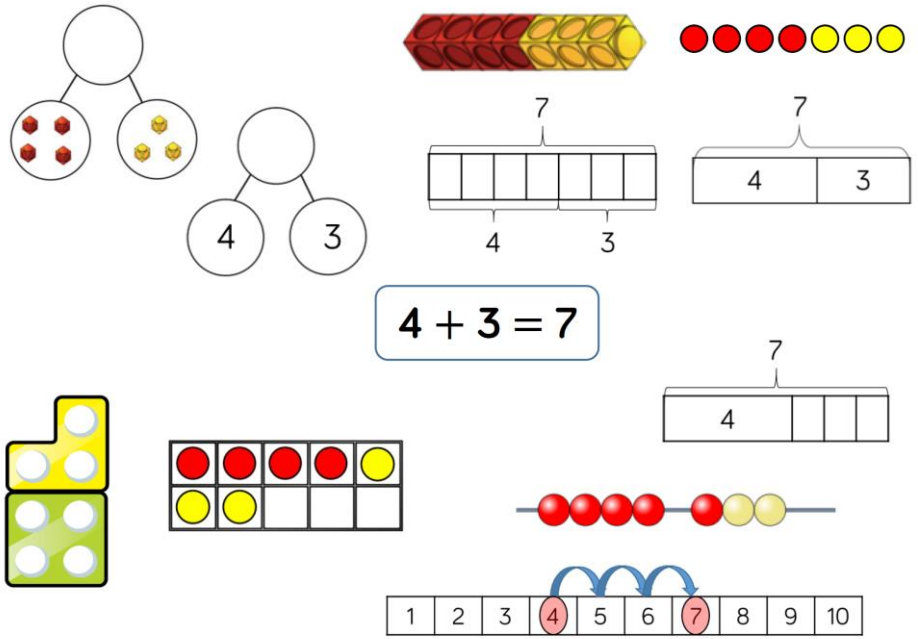
Near doubles

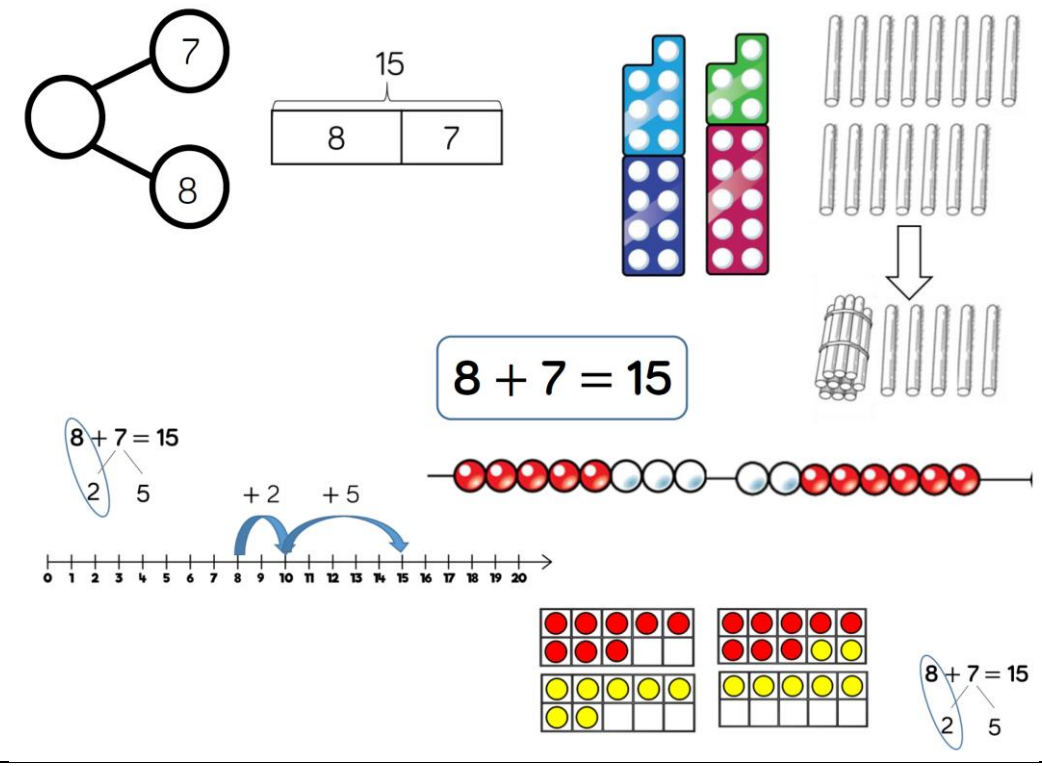
Multiplication and Division Facts

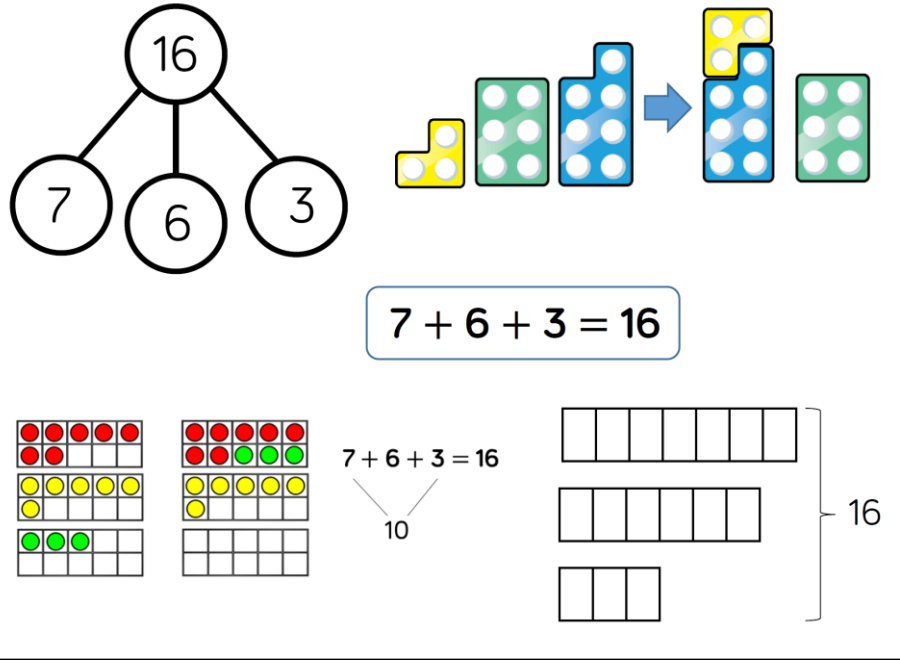
Times table facts up to 12 x 12 are learned in Years 3 & 4 and practiced in Years 5 & 6. These are the 36 facts that all children need to automatically recall by the end of Year 4 in order to be ready for short multiplication and division strategies in Year 5:

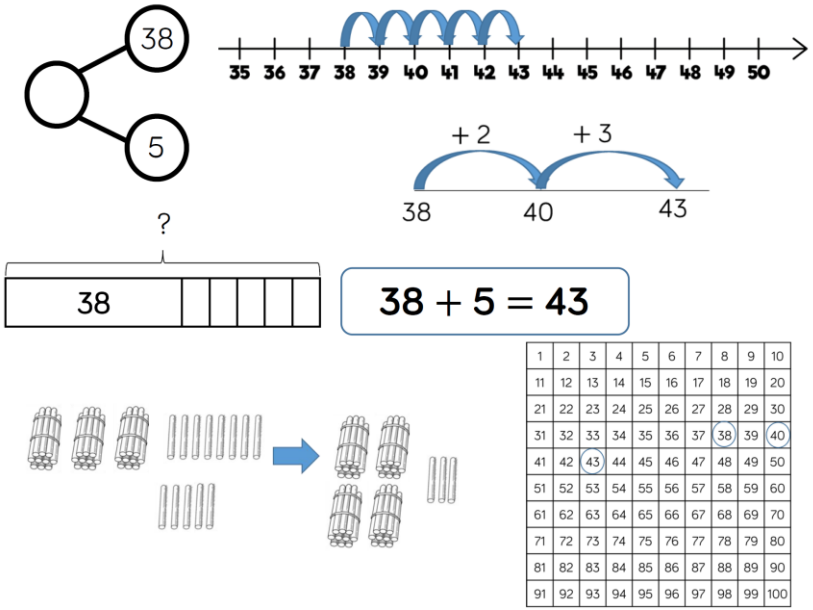
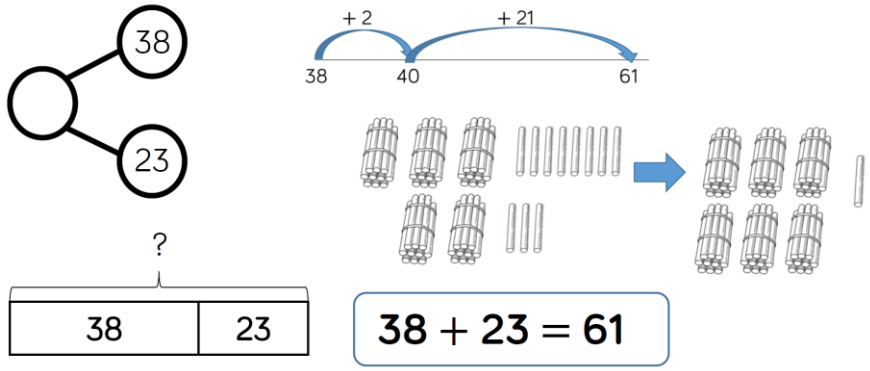
2 times tables	3 times tables	4 times tables	5 times tables	6 times tables	7 times tables	8 times tables	9 times tables
2 x 2 = 4							
3 x 2 = 6	3 x 3 = 9						
4 x 2 = 8	4 x 3 = 12	4 x 4 = 16					
5 x 2 = 10	5 x 3 = 15	5 x 4 = 20	5 x 5 = 25				
6 x 2 = 12	6 x 3 = 18	6 x 4 = 24	6 x 5 = 30	6 x 6 = 36			
7 x 2 = 14	7 x 3 = 21	7 x 4 = 28	7 x 5 = 35	7 x 6 = 42	7 x 7 = 49		
8 x 2 = 16	8 x 3 = 24	8 x 4 = 32	8 x 5 = 40	8 x 6 = 48	8 x 7 = 56	8 x 8 = 64	
9 x 2 = 18	9 x 3 = 27	9 x 4 = 36	9 x 5 = 45	9 x 6 = 54	9 x 7 = 63	9 x 8 = 72	9 x 9 = 81

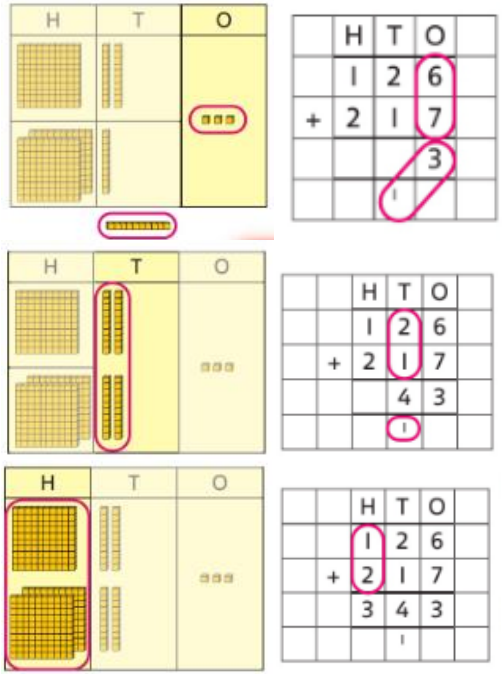
Addition	Representations	Notes
Reception Combining two parts to make a whole (aggregation)	 (use other resources too e.g. eggs, shells, teddy bears, cars).  Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.	**4 + 3 = 7** **7 = 4 + 3** Four is a part, three is a part and the whole is seven. 

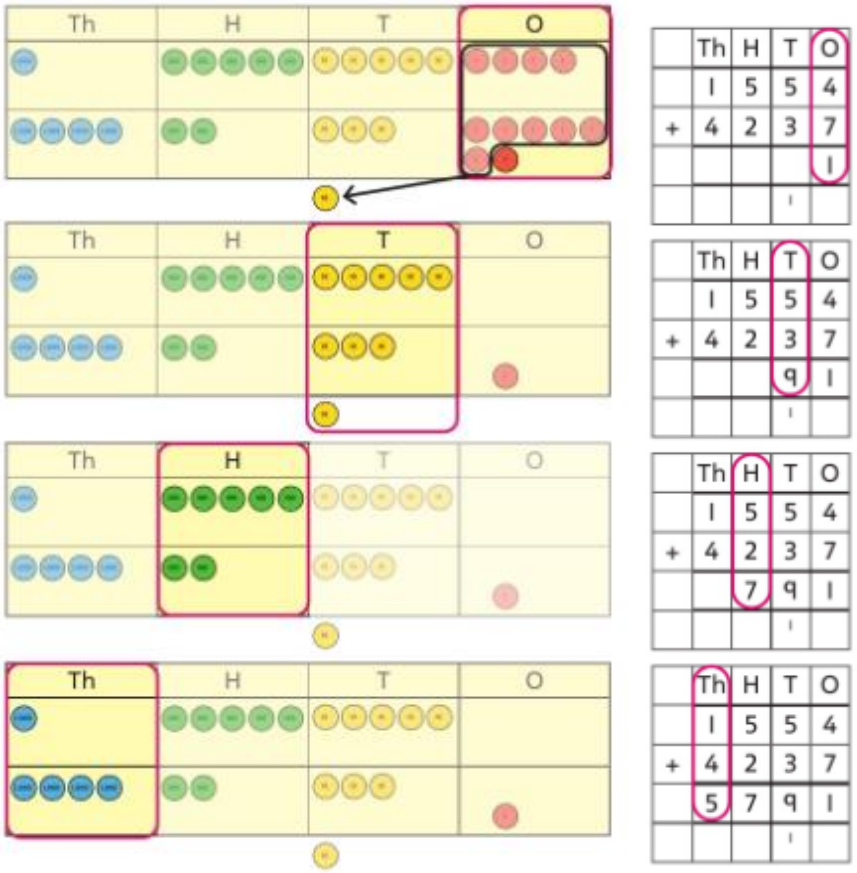
Addition	Representations	Notes
Year 1 Add 1-digit numbers within 10		When adding numbers to 10, children can explore both aggregation and augmentation. The part whole model, discrete and continuous bar model, number shapes and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.

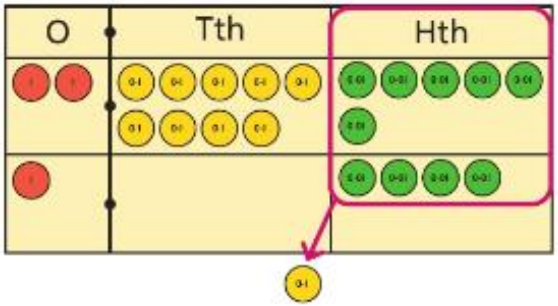
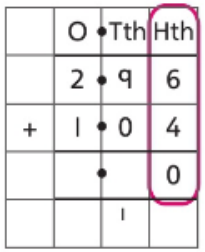
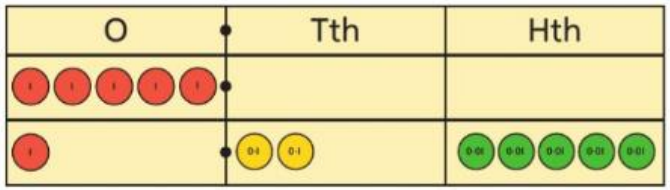
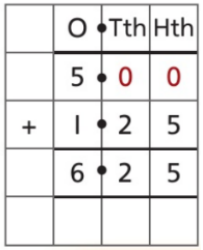
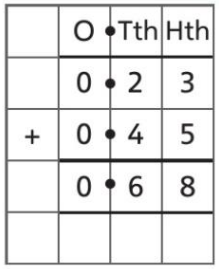
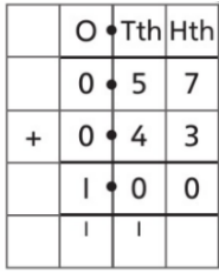
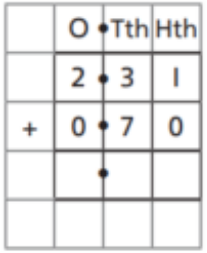
Addition	Representations	Notes
Year 1 & 2 Add 1 and 2 digit numbers to 20		When adding one digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. In Year 1, this is only done just by counting on. From Year 2, use different manipulatives can be used to represent this exchange alongside number lines to support children in understanding how to partition their jumps

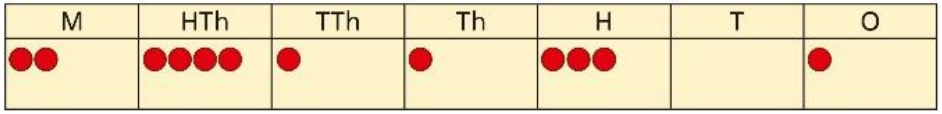
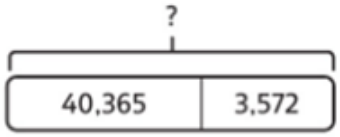
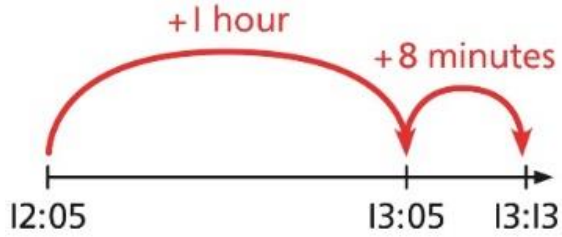
Addition	Representations	Notes
Year 2 Add three 1-digit numbers		When adding three 1 digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1 digit numbers.

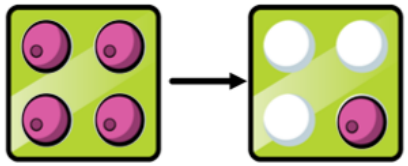
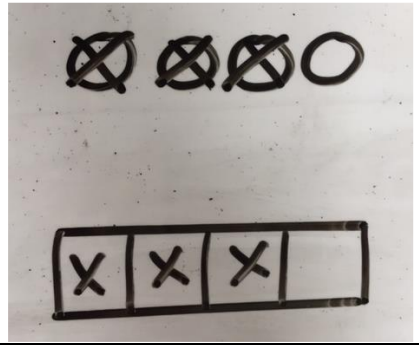
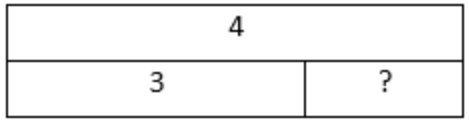
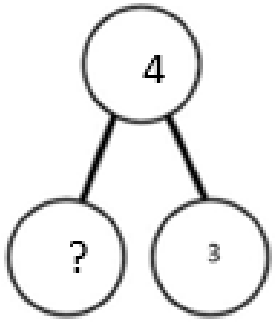
Addition	Representations	Notes
Year 2 Add 1 digit and 2-digit numbers to 100		When adding single digits to a two digit number, children should be encouraged to count on from the larger number. They should also apply their knowledge of number bonds to add more efficiently e.g. 8 +5 =13 so 38 +5 =43. Hundred squares and straws can support children to find the number bond to 10.
Year 2 Add two 2-digit numbers to 100		Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

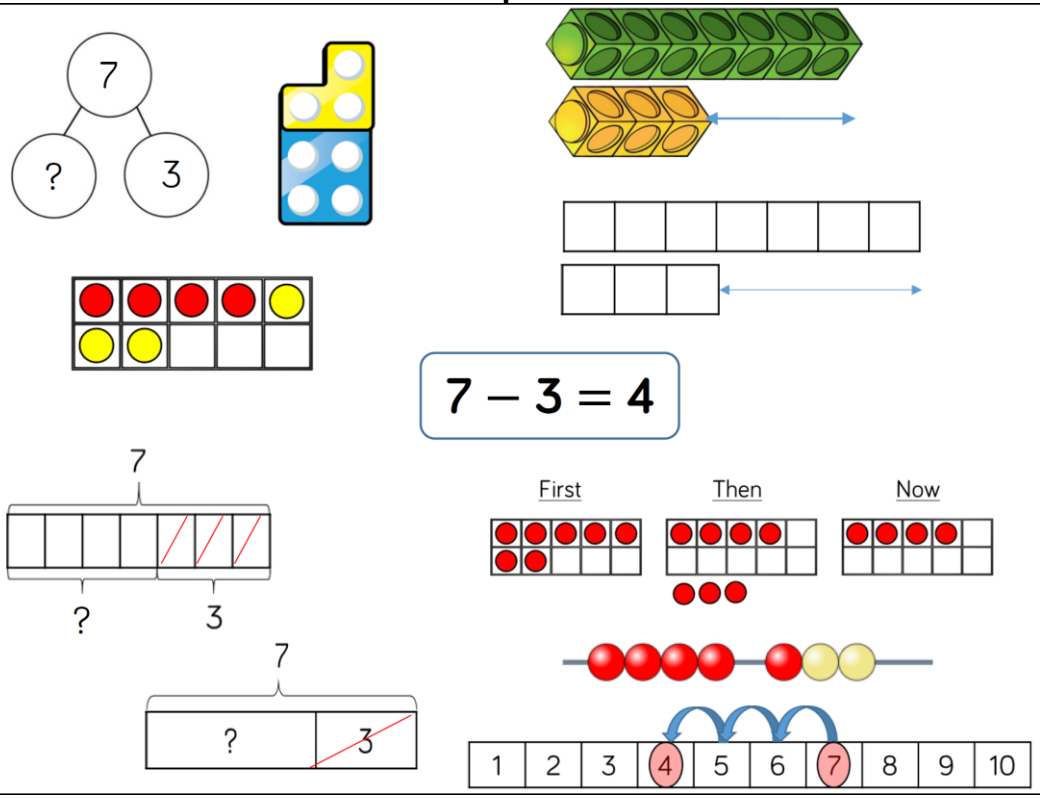
Addition	Representations	Notes
Year 3 Column addition with exchange		Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.

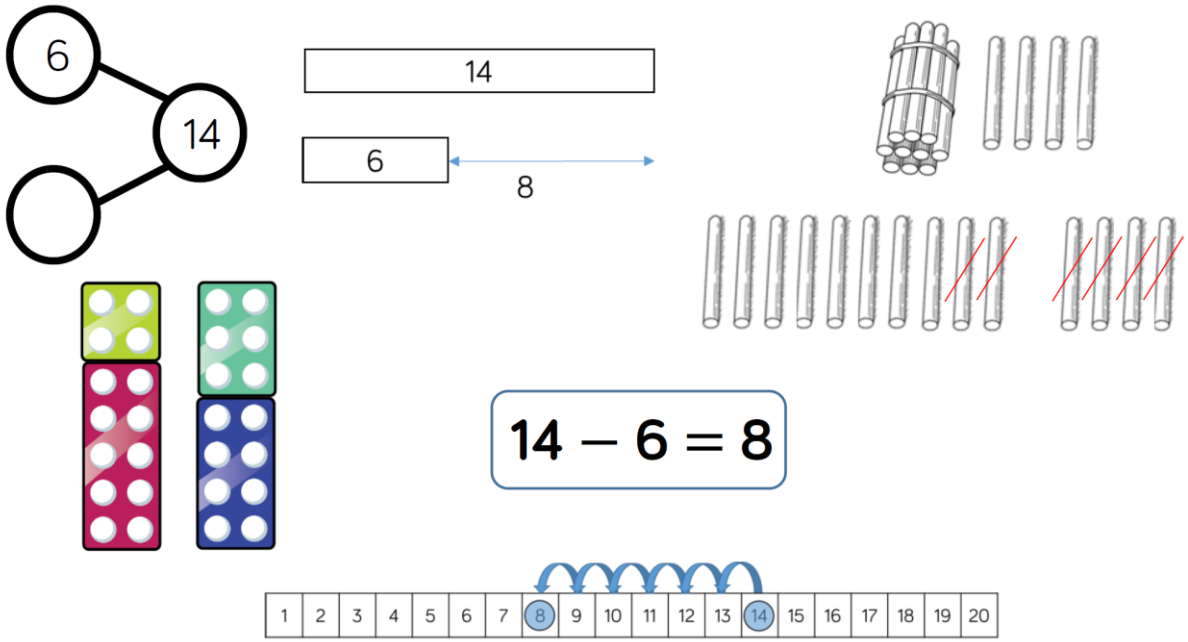
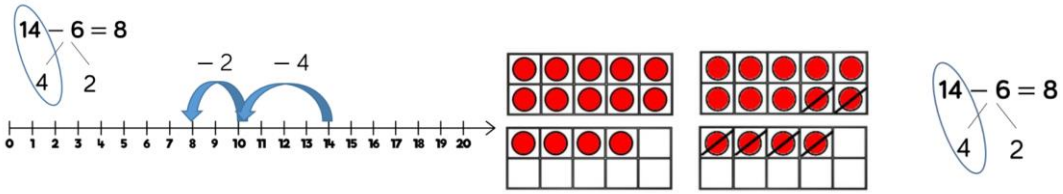
Addition	Representations	Notes
Year 4 Column layout-exchanging. (up to 4 digits)	**Place Value Counters** on Grids – Thousands, Hundreds, Tens and Units  Th H T O 1 5 5 4 + 4 2 3 7 1 Th H T O 1 5 5 4 + 4 2 3 7 9 1 Th H T O 1 5 5 4 + 4 2 3 7 7 9 1 Th H T O 1 5 5 4 + 4 2 3 7 5 7 9 1	**Compact column addition** with larger numbers

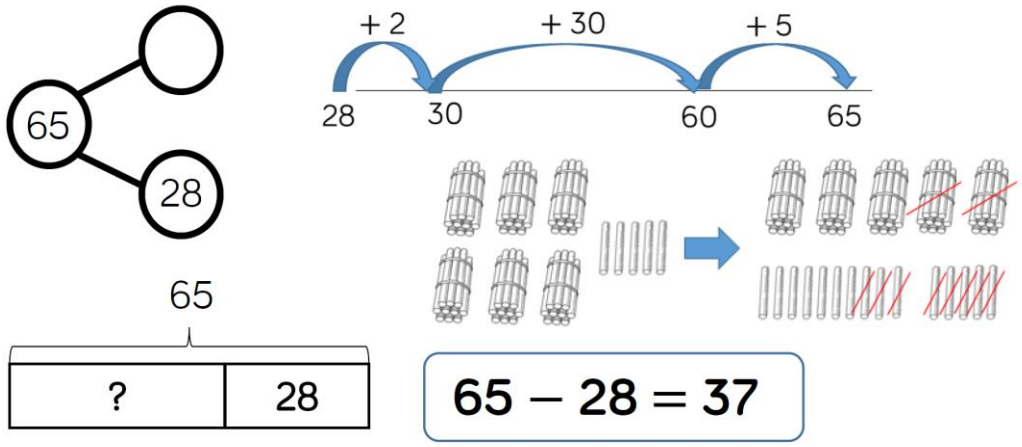
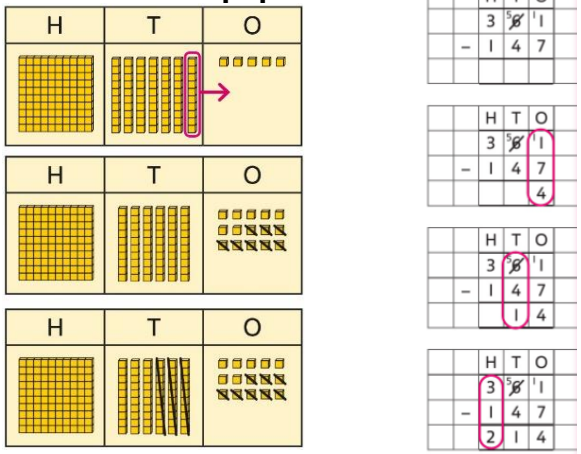
Addition	Representations	Notes
Year 5 Column layout- exchanging (including decimals) .	Use place value equipment on a place value grid to represent additions. Represent exchange where necessary.   Include examples where the numbers of decimal places are different.   	Add using a column method, ensuring that children understand the link with place value.  Include exchange where required, alongside an understanding of place value.  Include additions where the numbers of decimal places are different. 

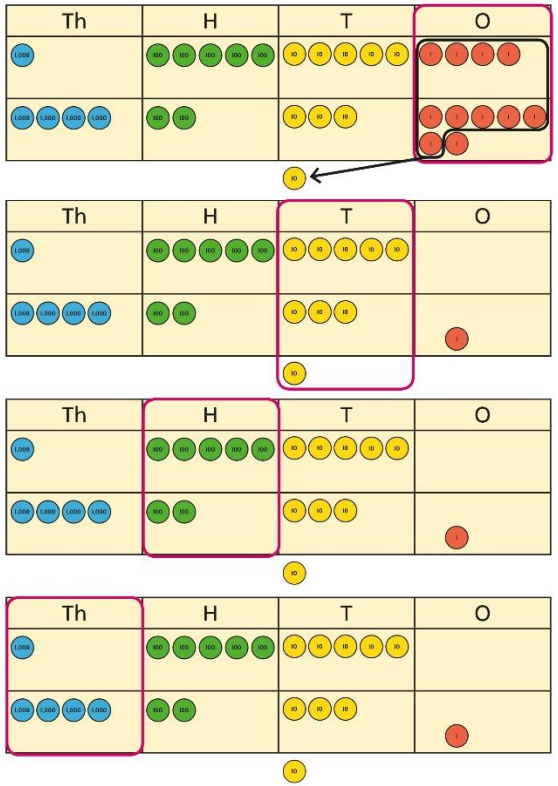
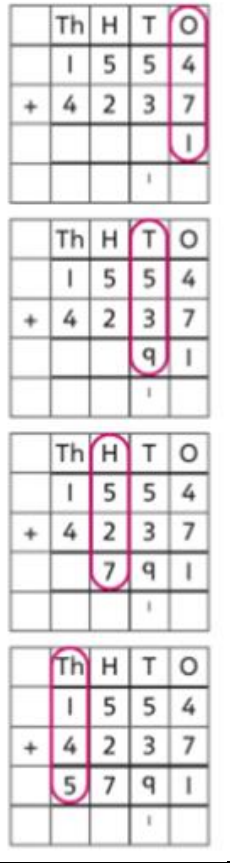
Addition	Representations	Notes
Year 6 Comparing and selecting efficient methods	Represent 7-digit numbers on a place value grid and use this to support thinking and mental methods.  Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.  TTh Th H T O 4 0 3 6 5 + 3 5 7 2 Use bar model and number line representations to model addition in problem-solving and measure contexts.  	Use column addition where mental methods are not efficient. Recognise common errors with column addition. *17,877 + 4,012 = ?* TTh Th H T O 1 7 8 7 7 + 4 0 1 2 5 7 9 9 7 TTh Th H T O 1 7 8 7 7 + 4 0 1 2 2 1 8 8 9 *Which method has been completed accurately?* *What mistake has been made?* Column methods are also used for decimal additions where mental methods are not efficient. H T O • Tth Hth 1 4 0 • 0 9 + 4 9 • 8 9 1 8 9 • 9 8 1

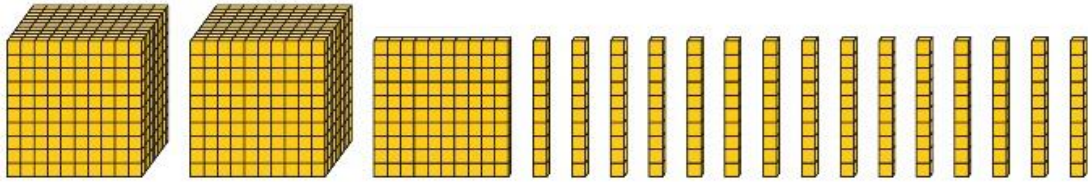
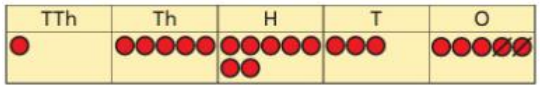
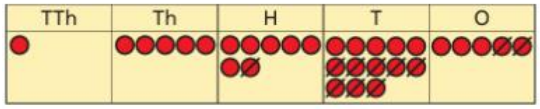
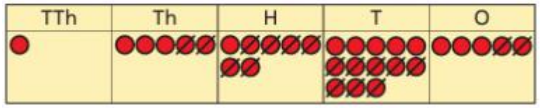
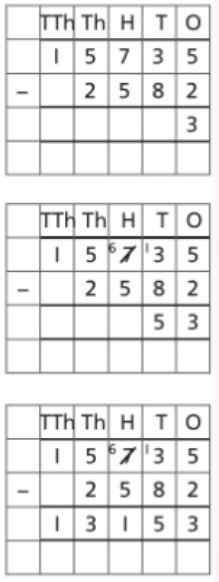
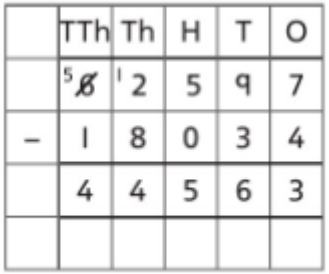
Subtraction	Representations	Notes
Reception Subtraction by taking away	Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1   Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = ☐ ☐ = 4 – 3  

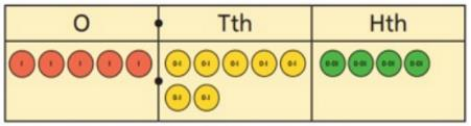
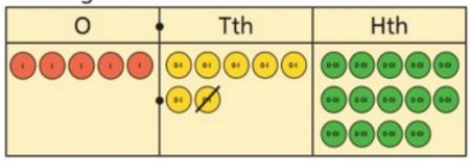
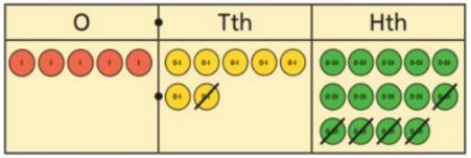
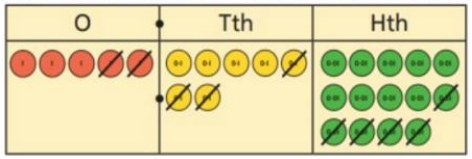
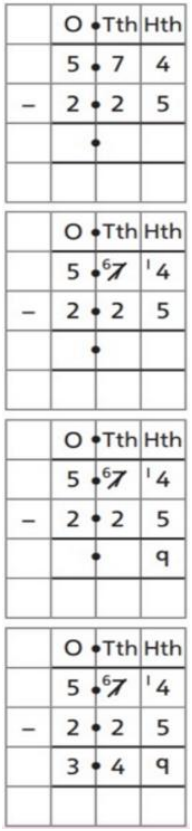
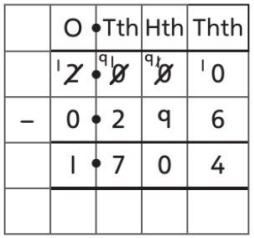
Subtraction	Representations	Notes
Year 1 Subtract 1 digit numbers within 10		Part whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.

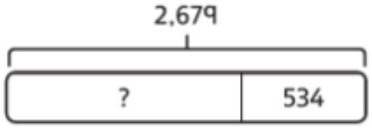
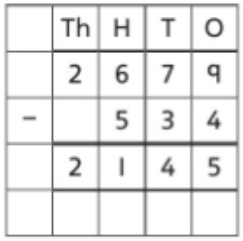
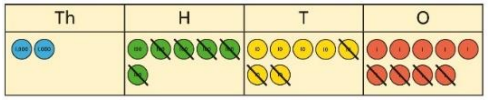
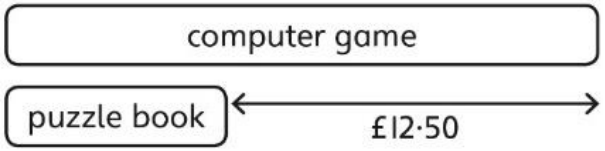
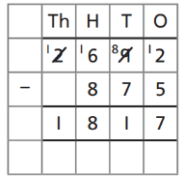
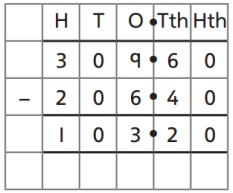
Subtraction	Representations	Notes
Year 1 Subtract 1 and 2 digit numbers to 20		In Year 1, subtracting one digit numbers that cross 10, is done by counting back, using objects, number tracks and number lines.
Year 2 Subtract 1 and 2 digit numbers to 20		From Year 2, children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

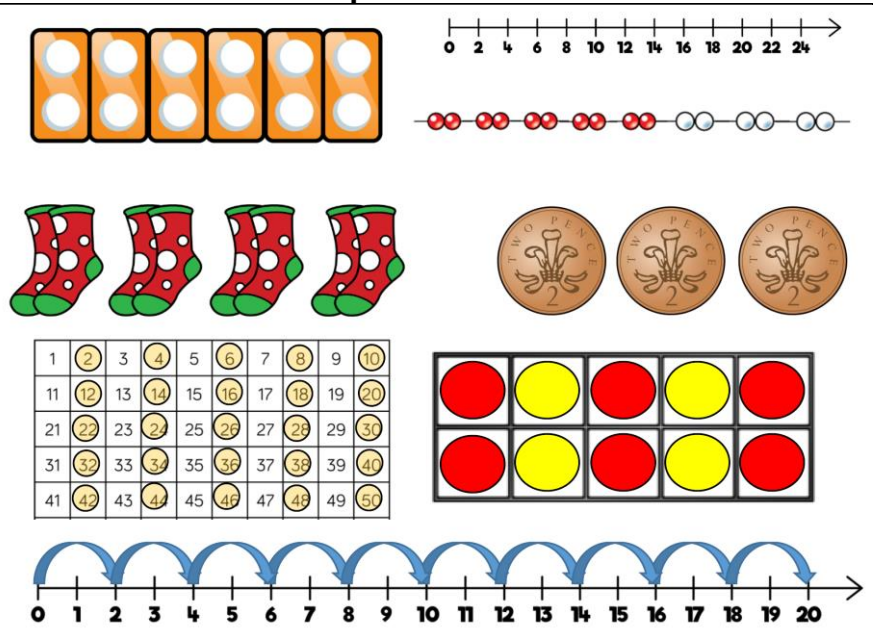
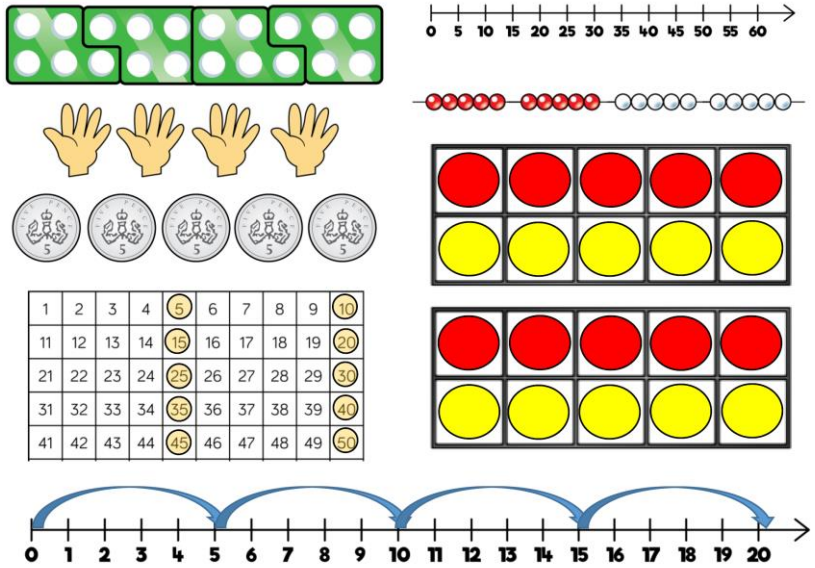
Subtraction	Representations	Notes
Year 2 Subtract 1 and 2 digit numbers to 100		Children can also use a blank number line to count back to find the difference. Encourage them to jump to multiples of 10 to become more efficient.
Year 3 Column layout with exchanging (up to 3 digits)	**Place Value Equipment**  	Children should also understand how to exchange in calculations where there is a zero in the 10s column.

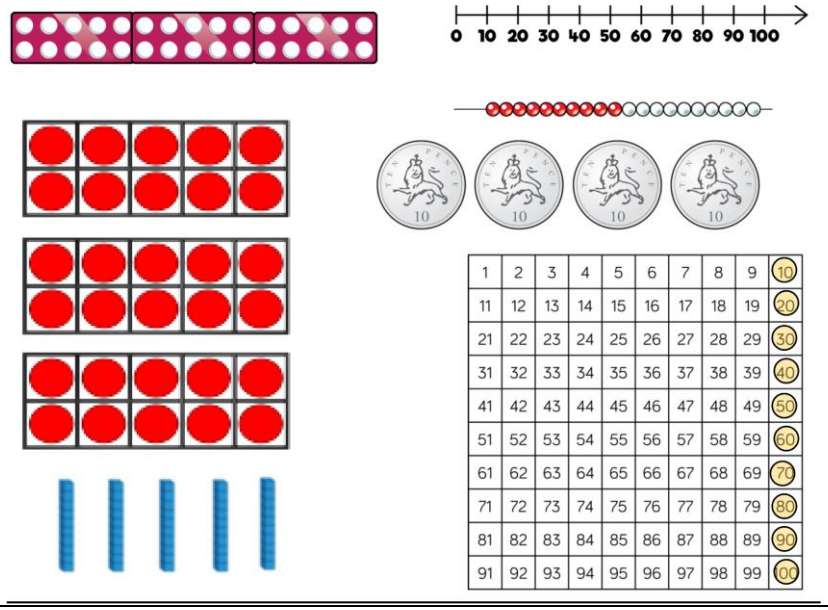
Subtraction	Representations	Notes
Year 4 Column subtraction with exchanging (up to 4 digits).	**Place Value Counters** on Grids – Thousands, Hundreds, Tens and Units. Children may need to exchange more than once.   	Also develop compact column subtraction with more than one exchange

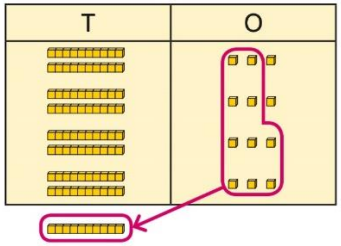
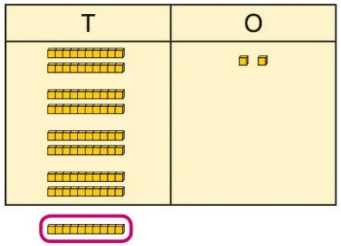
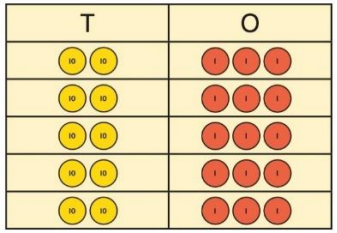
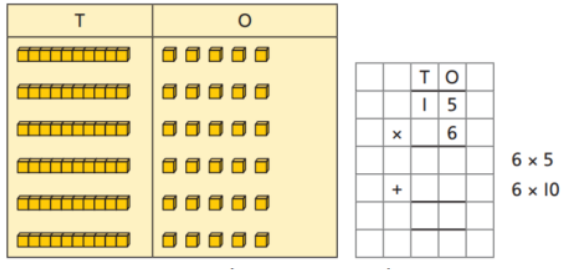
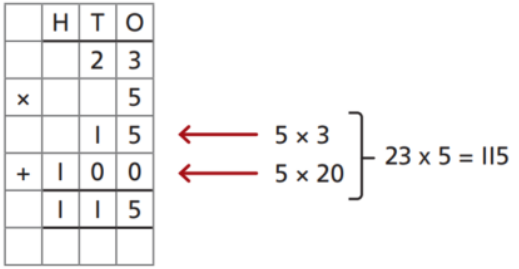
Subtraction	Representations	Notes
Year 5 Column subtraction with whole numbers where exchanges are required.	*2,250 – 1,070 = ?*  Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. *15,735 − 2,582 = 13,153*  Now subtract the 10s. Exchange 1 hundred for 10 tens.  Subtract the 100s, 1,000s and 10,000s.  	Use column subtraction methods with exchange where required.  62,597 − 18,034 = 44,563

Subtraction	Representations	Notes
Year 5 Column subtraction with exchanging (2-place decimals)	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 − 2·25 = ?*  Exchange 1 tenth for 10 hundredths.  Now subtract the 5 hundredths.  Now subtract the 2 tenths, then the 2 ones.   	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *2·000 − 0·296 = ?* 

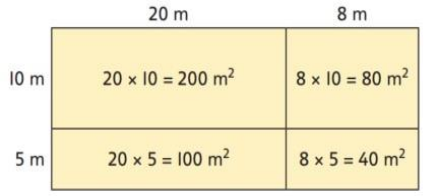
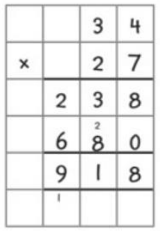
Subtraction	Representations	Notes
Year 6 Comparing and selecting efficient methods	Compare subtraction methods alongside place value representations.    Use a bar model to represent calculations, including ‘find the difference’ with two bars as comparison. 	Compare and select methods. Use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy.  Use column subtraction for decimal problems, including in the context of measure. 

Multiplication	Representations	Notes
Year 2 2 times table		Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones. Use different models to develop fluency.
Year 2 5 times table		Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

Multiplication	Representations	Notes
Year 2 10 times table		Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits the ones are always 0, and the tens increase by 1 ten each time.

Multiplication	Representations	Notes
Year 3 Column layout to TU x U	Using place value equipment (with exchanging)   *4 × 23 = 92* Children to represent the counters/base 10, pictorially e.g. the image below.  *5 × 23 = ?* *5 × 3 = 15* *5 × 20 = 100* *5 × 23 = 115*	Formal written method  

Multiplication	Representations	Notes
Year 4 Column multiplication (2 and 3 digit multiplied by 1 digit)	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  H T O 3 1 2 × 3 9 3 6	Use the formal column method for up to 3-digit numbers multiplied by a single digit. H T O 3 1 2 × 3 9 3 6 Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. H T O 2 3 × 5 1 5 + 1 0 0 1 1 5 H T O 2 3 × 5 1 1 5 1

Multiplication	Representations	Notes
Year 5 Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?*  10 × 15 = 150 10 × 15 = 150  3 × 15 = 45 There are 345 bottles of milk in total. H T O 1 5 0 1 5 0 + 4 5 3 4 5 1 *23 × 15 = 345* Use an area model and add the parts. *28 × 15 = ?*  20 m 8 m 10 m: 20 × 10 = 200 m²	8 × 10 = 80 m² 5 m: 20 × 5 = 100 m²	8 × 5 = 40 m² H T O 2 0 0 1 0 0 8 0 + 4 0 4 2 0 1 *28 × 15 = 420*	Use column multiplication, ensuring understanding of place value at each stage.  3 4 × 2 7 2 3 8 6 8 0 (2) 9 1 8 1 34 × 7 34 × 20 34 × 27

Year 5
Multiplying up to 4-digits by 2-digits

Use the area model then add the parts.

	100	40	3
10	100 × 10 = 1,000	40 × 10 = 400	3 × 10 = 30
2	100 × 2 = 200	40 × 2 = 80	3 × 2 = 6

	Th	H	T	O
	1	0	0	0
		4	0	0
		2	0	0
			8	0
			3	0
+				6
	1	7	1	6
		1		

143 × 12 = 1,716

Use column multiplication, ensuring understanding of place value at each stage.

	Th	H	T	O	
		1	4	3	
×			1	2	
		2	8	6	143 × 2
	1	4	3	0	143 × 10
	1	7	1	6	143 × 12
		1			

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

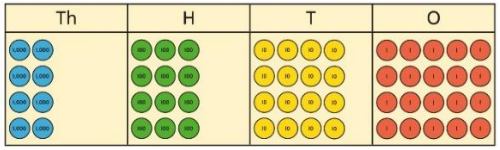
1,274 × 32 = ?

First multiply 1,274 by 2.

	TTh	Th	H	T	O	
		1	2	7	4	
×				3	2	
		2	5	4	8	1,274 × 2
			1			

Then multiply 1,274 by 30.

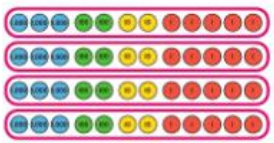
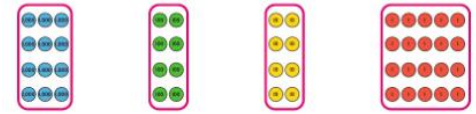
	TTh	Th	H	T	O	
		1	2	7	4	
×				3	2	
		2	5	4	8	1,274 × 2
	3	8	1 2	2	0	1,274 × 30
		2	1			

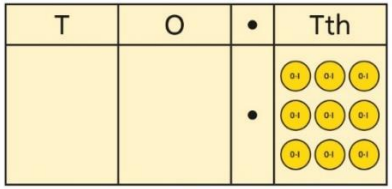
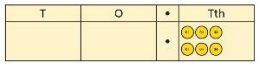
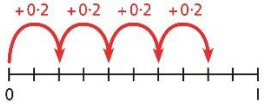
<table>
<tr><th>Multiplication</th><th>Representations</th><th>Notes</th><th></th></tr>
<tr><td></td><td></td><td>Finally add up the numbers.
<table><tr><td></td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td>7</td><td>4</td></tr><tr><td>×</td><td></td><td></td><td></td><td>3</td><td>2</td></tr><tr><td></td><td></td><td>2</td><td>5</td><td>4</td><td>8</td></tr><tr><td></td><td>3</td><td>8</td><td>1
2</td><td>2</td><td>0</td></tr><tr><td></td><td>4</td><td>2
0</td><td>1
7</td><td>6</td><td>8</td></tr><tr><td></td><td>1</td><td></td><td></td><td></td><td></td></tr></table>1,274 × 2
1,274 × 30
1,274 × 32</td><td></td></tr>
<tr><td>Year 6
Multiplying up to a 4-digit number by a single digit number</td><td>Use equipment to explore multiplications.
 

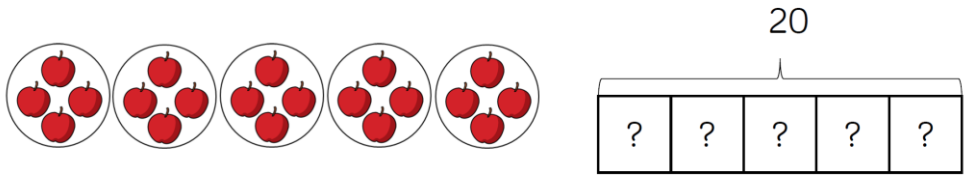
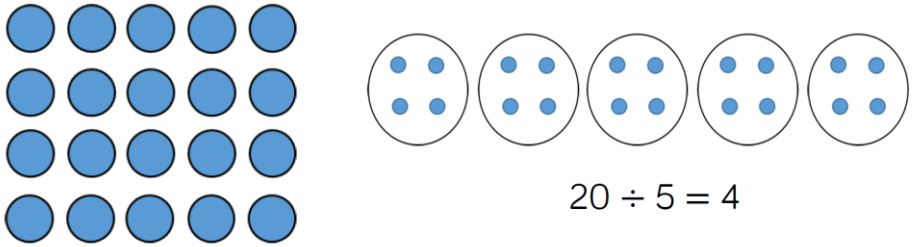
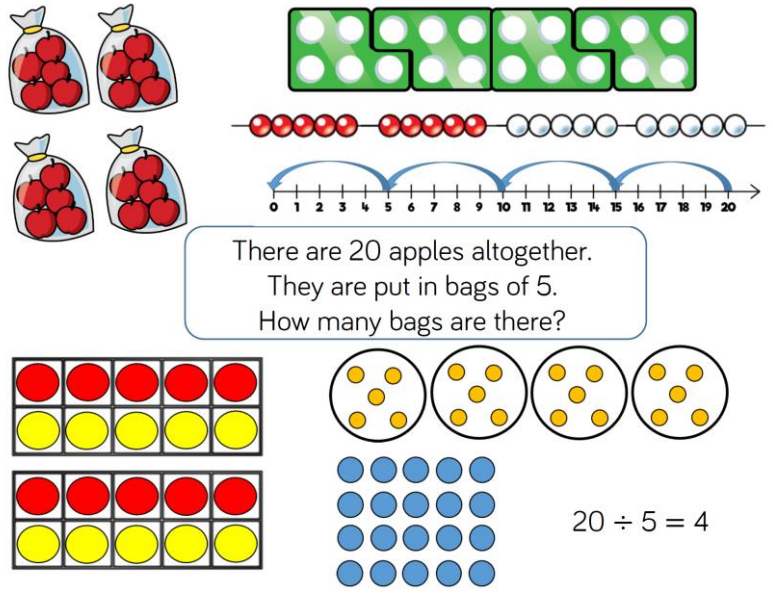
4 groups of 2,345

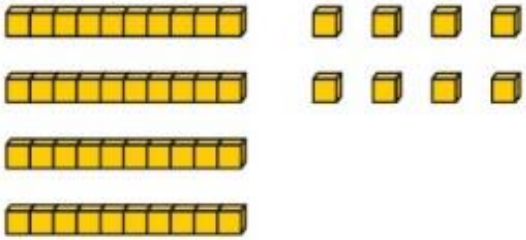
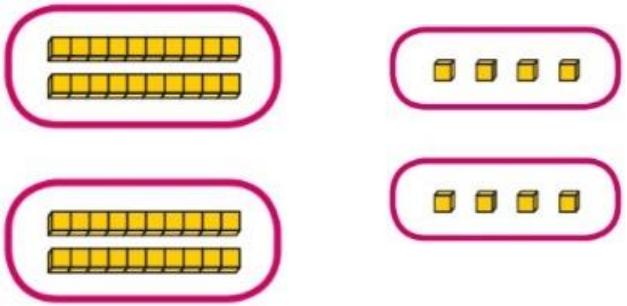
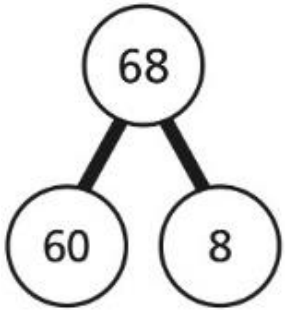
This is a multiplication:

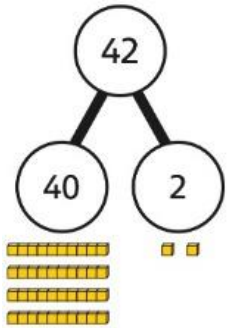
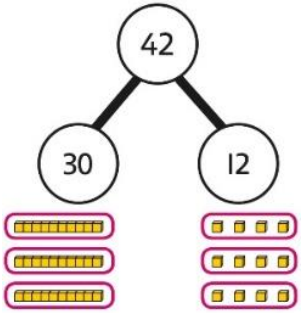
4 × 2,345
2,345 × 4</td><td>Understand area model and short multiplication.
Compare and select appropriate methods for specific multiplications.
Method 3
<table><tr><td></td><td>3,000</td><td>200</td><td>20</td><td>5</td></tr><tr><td>4</td><td>12,000</td><td>800</td><td>80</td><td>20</td></tr></table>12,000 + 800 + 80 + 20 = 12,900
Method 4
<table><tr><td></td><td></td><td>3</td><td>2</td><td>2</td><td>5</td></tr><tr><td>×</td><td></td><td></td><td></td><td></td><td>4</td></tr><tr><td></td><td>1</td><td>2</td><td>9</td><td>0</td><td>0</td></tr><tr><td></td><td>1</td><td></td><td>1</td><td>2</td><td></td></tr></table></td><td></td></tr>
</table>

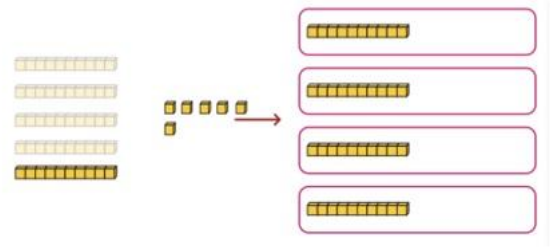
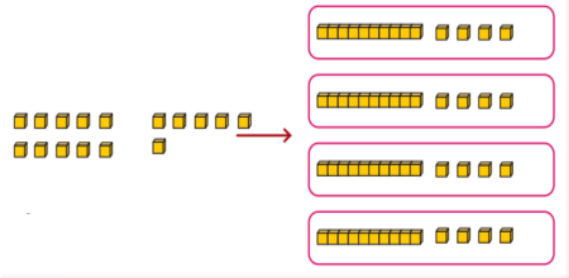
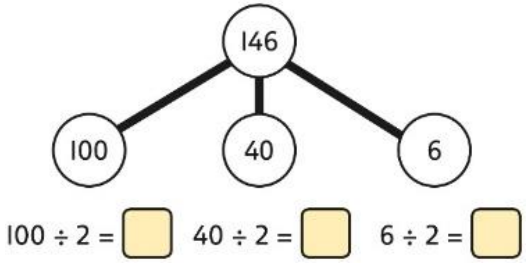
Multiplication	Representations	Notes
	Use place value equipment to compare methods. **Method 1**  + 3 2 5 5 3 2 2 5 3 2 2 5 3 2 2 5 1 2 9 0 0 1 1 1 **Method 2**  4 × 3,000 + 4 × 200 + 4 × 20 + 4 × 5 12,000 + 800 + 80 + 20 = 12,900	
Year 6 Multiplying up to a 4-digit number by a 2-digit number	200 30 5 20 4,000 600 100 1 200 30 5 4,200 + 630 + 105 = 4,935 Use an area model alongside written multiplication. 2 3 5 × 2 1 5 1 × 5 3 0 1 × 30 2 0 0 1 × 200 1 0 0 20 × 5 6 0 0 20 × 30 4 0 0 0 20 × 300 4 9 3 5 21 × 235	Use compact column multiplication with understanding of place value at all stages. 2 3 5 × 2 1 2 3 5 1 × 235 4 7 0 0 20 × 235 4 9 3 5 21 × 235

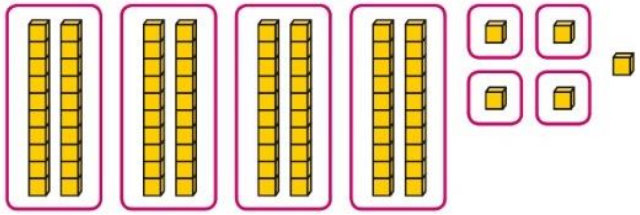
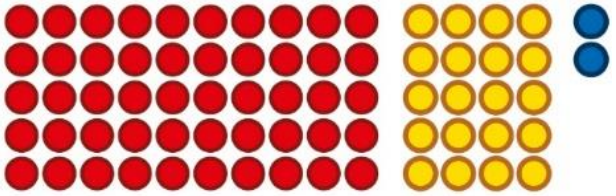
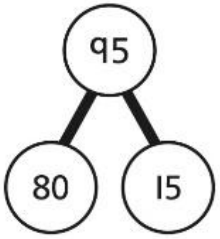
Multiplication	Representations	Notes
Year 6 Multiplying Decimals	Represent calculations on a place value grid. 3 × 3 = 9 3 × 0·3 = 0·9  Understand the link between multiplying decimals and repeated addition.   	Use known facts to multiply decimals. *4 × 3 = 12* *4 × 0·3 = 1·2* *4 × 0·03 = 0·12* *20 × 5 = 100* *20 × 0·5 = 10* *20 × 0·05 = 1* Find families of facts from a known multiplication. *I know that 18 × 4 = 72.* *This can help me work out:* *1·8 × 4 = ?* *18 × 0·4 = ?* *180 × 0·4 = ?* *18 × 0·04 = ?* Use a place value grid to understand the effects of multiplying decimals. H T O • Tth Hth 2 × 3: 6 • 0·2 × 3: 0 • 6 0·02 × 3: •

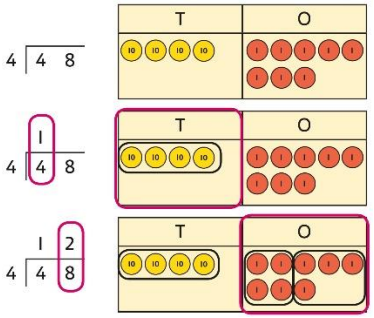
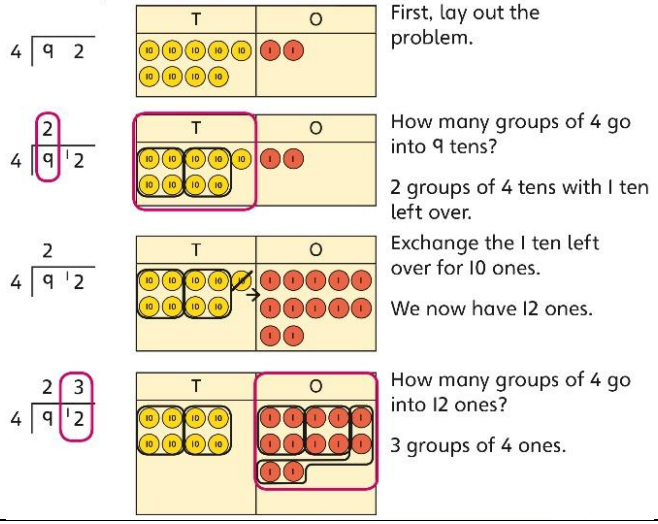
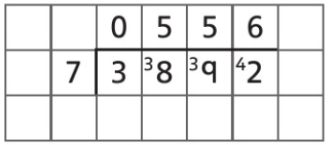
Division	Representations	Notes
Year 1 Sharing objects equally into groups		Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.
Year 2 Division as sharing		In Year 2, children are introduced to the division symbol.
Year 2 Division as grouping (repeated addition)		Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

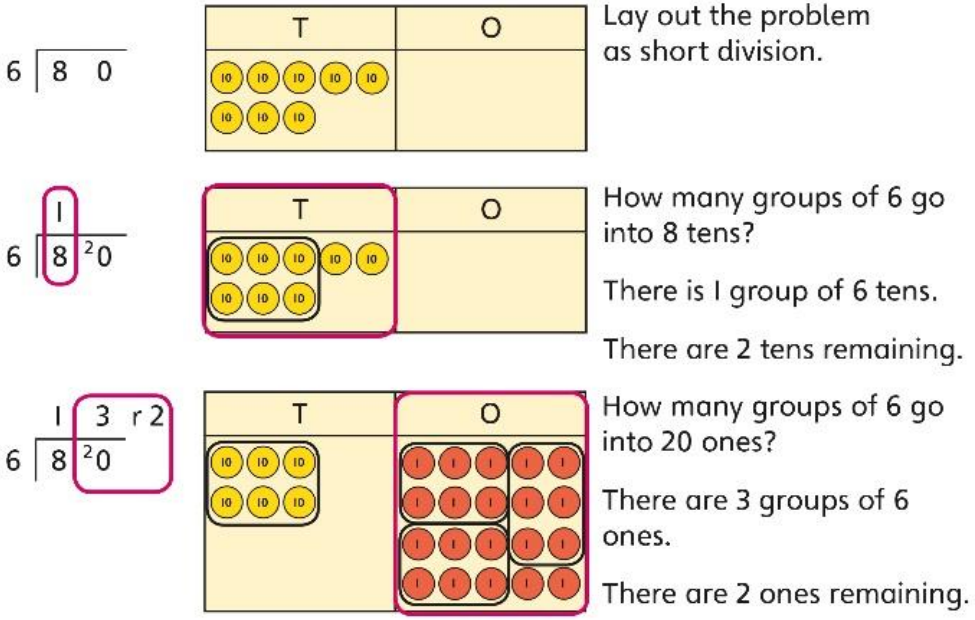
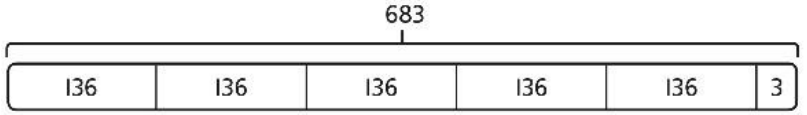
Division	Representations	Notes
Year 3 2-digit number divided by 1-digit number	Children explore dividing 2-digit numbers by using place value equipment.  *48 ÷ 2 = ?* *First divide the 10s.* *Then divide the 1s* 	Children partition a number into 10s and 1s to divide where appropriate.  *60 ÷ 2 = 30* *8 ÷ 2 = 4* *68 ÷ 2 = 34*

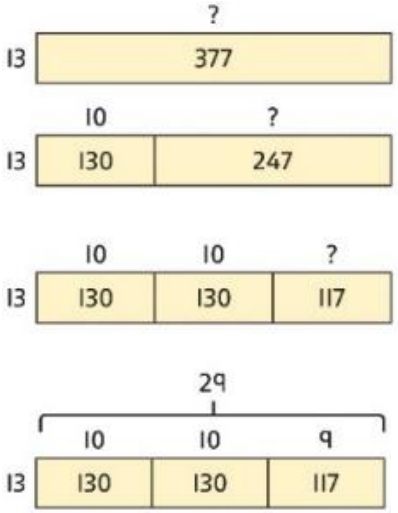
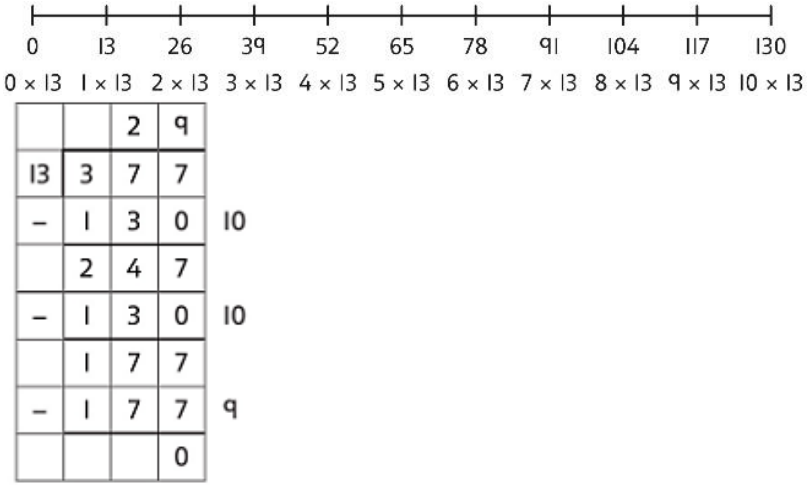
Division	Representations	Notes
Year 3 Divide with partitioning	Children explore which partitions support particular divisions.  *I need to partition 42 differently to divide by 3.*  *42 = 30 + 12* *42 ÷ 3 = 14*	Children partition flexibly to divide where appropriate. *42 ÷ 3 = ?* *42 = 40 + 2* *I need to partition 42 differently to divide by 3.* *42 = 30 + 12* *30 ÷ 3 = 10* *12 ÷ 3 = 4* *10 + 4 = 14* *42 ÷ 3 = 14*

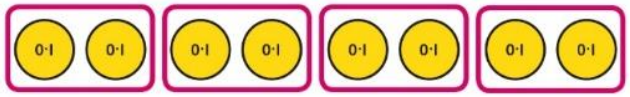
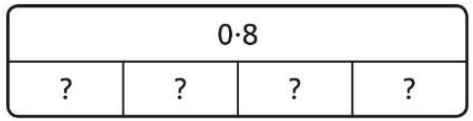
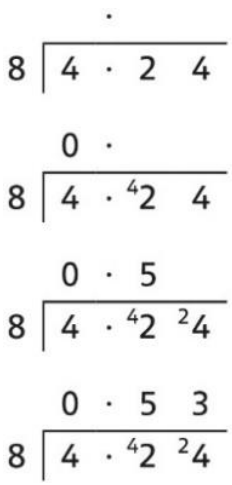
Division	Representations	Notes
Year 4 Divide by sharing with exchange	Share by exchanging *56 shared equally between 4 groups* *First share the 10s.*  *Exchange 1 ten for 1s, then share all the 1s.*  56 ÷ *4 = 14*	Share using known facts and partitioning where appropriate *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* 142 ÷ 2 = 73

Division	Representations	Notes
Year 4 Understand remainders	Use place value equipment to find remainders. *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.*  Represent the remainder as the part that cannot be shared equally.  72 ÷ 5 = 14 remainder 2	Understand how partitioning can reveal remainders of divisions.  *80 ÷ 4 = 20* *12 ÷ 4 = 3* 95 ÷ 4 = 23 remainder 3

Division	Representations	Notes
Year 5 Short division (up to 4 digits by a 1 digit number including remainders)	Use place value equipment on a place value grid alongside short division. The model uses grouping. **A sharing model can also be used, although the model would need adapting.**  Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* Work with divisions that require exchange.  	Use short division for up to 4-digit numbers divided by a single digit.  *3,892 ÷ 7 = 556* Use multiplication to check. *556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*

Division	Representations	Notes
Year 5 Find remainders	Understand remainders using concrete versions of a problem. *80 cakes divided into trays of 6.*  *80 cakes in total. They make 13 groups of 6, with 2 remaining.* Use short division and understand remainders as the last remaining 1s. 	In problem solving contexts, represent divisions including remainders with a bar model.  *683 = 136 × 5 + 3* *683 ÷ 5 = 136 r 3*

Division	Representations	Notes
Year 6 Dividing by a 2-digit number using long division	Use an area model alongside written division to model the process. *377 ÷ 13 = ?*  *377 ÷ 13 = 29*	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. *377 ÷ 13 = ?*  *377 ÷ 13 = 29*

Division	Representations	Notes
Year 6 Divide decimals by short division	Use place value equipment to explore division of decimals.  *8 tenths divided into 4 groups. 2 tenths in each group.* Use a bar model to represent divisions.  4 × 2 = 8 8 ÷ 4 = 2 So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2	Use short division to divide decimals with up to 2 decimal places. 

[this page is unintentionally blank]