ESW Primary maths calculation policy

Aims:

To explain our main methods for teaching arithmetic (addition, subtraction, multiplication, and division).

To reduce confusion by making sure that all adults working with our pupils use the same methods.

Contents:

Notes:

This document outlines the main approaches we expect all of our pupils to be familiar with. However, there are other representations and methods that might be used by teachers for specific reasons.

General principles

Long term utility of methods

We aim to identify and avoid teaching things that lead pupils into long term misconceptions. For example, multiplying by 10 can be done by adding a zero (0.8 x 10 = 0.80), or multiplying makes larger (6 x 0.5 = 3)

Partitioning

Our main representation for partitioning is the bar model. This is because it is used to support understanding in many other topic areas including ratio and fractions.

The whole-part-whole model which uses connected circles does not have wider application and also is easily confused with prime factor trees used in KS3 for prime factor decomposition.

Moving children away from counting strategies

- Obvious use of fingers to count on indicates a pupil either:
  - does not have fluent recall of relevant facts (multiplication/number bonds) or
  - does not know how to apply the facts they do have
- Pupils regularly using fingers to count on are assessed to identify the reason and appropriate follow up work is set.
- Pupils know (because they are regularly told) that manipulatives and diagrams are used to help them do maths in their heads. They are a temporary stage (this does not apply to written methods).
- The use of manipulatives (or pen/paper for number line diagrams) is deliberately withdrawn when the teacher believes pupils are ready to visualise calculation mentally.
- Manipulatives are used to recap, repair to support pupils when explaining their thinking.

Intelligent practice and variation

When selecting (or writing) questions for pupil practice the aim is to ensure that

- the appropriate skill is practiced,
- the structure of calculations is exposed so that pupils can form generalisations,
- pupils meet special cases of calculations.

Numbers chosen for practice are important. Random or unplanned approaches to question creation (for example the use of dice) should be avoided.

Questions should be written using numbers that ensure that pupils practice the desired skills. For example:

- 123 + 432 would not give a pupil practice of exchanging.
- 21 + 19 could be used to find pupils who do not know how to use number bonds.
- 12 x 25 is appropriate for a child who does not know their 7 times table. 67 x 87 is not.
- 728 - 20 would not help a child practicing the written column method.
- 237 x 11 would reduce cognitive load when learning the long multiplication algorithm as pupils would only need to multiply by 1.

Format of calculations and missing number problems

Missing number problems form the basis of early algebra and it is common for pupils to struggle with them.

It is important that pupils regularly experience calculations written in different forms. The position of the unknown should be varied in teaching and practice. For example:

a) $3 \times 5 = \square$
b) $\square \times 5 = 15$
c) $3 \times \square = 15$
d) $3 \square 5 = 8$
e) $\square \times 5 = 2 \times 10$

By regularly varying the form of calculations, we help pupils to understand that the equality sign is not an instruction to ‘work out’, ‘evaluate’ or ‘find the answer’.

Mental Addition

Addition: mental addition of small numbers within 10 or 20

Example questions

a) $3 + 6 = 9, 9 = 3 + 6\ and\ 9 = 6 + 3$
b) $3 + 8$
c) $13 + 4$
d) $8 + 7 = 8 + 2 + 5$
e) $1 + 16 = 16 + 1$
f) Find the **sum** of 4 and 7
g) What is the **total** of 6 and 11?
h) **Add** 6 and 7

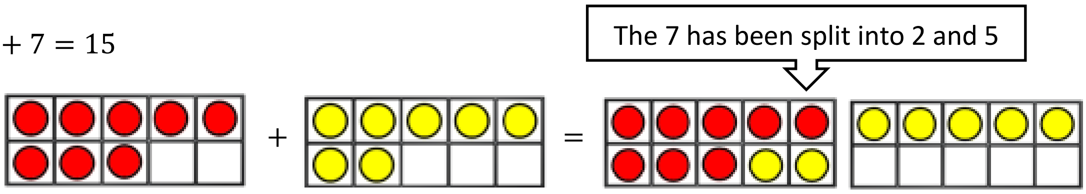
Barriers to success

Regularly assess pupils for:

- Lack of fluency with number bonds.
- Use of number bonds to 10 to cross 10 mentally.
- The ability to add two single digit numbers quickly
- Use of the associative and commutative properties of addition to speed up mental addition.
- A reliance on counting on fingers.
- A good knowledge of the words for addition.

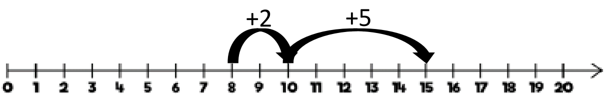
Tens frames are used with counters. Pupils can see how number bonds help with problems that **cross 10**.

Example: $8 + 7 = 15$



Number lines are used to show mental calculations with larger numbers.

Example: $8 + 7 = 15$



Addition: mental addition of numbers with two or more digits

Example questions

a) $43 + 6$
b) $23 + 24$
c) $273 + 10$
d) $73 + 22$
e) $3000 + 5000$
f) $54 + 9 + 6$
g) $199 + 27$

Barriers to success

As well as the areas listed above, assess pupils for:

- The ability to change the order of addition to simplify a calculation.
- The use of partitioning to simplify calculations.
- Over reliance on written methods without first considering more efficient approaches.

Mental methods

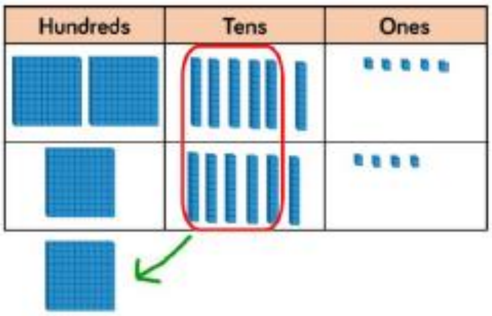
Adding from left to right	$43 + 32$ could be done by calculating $43 + 30 + 2$
Adding a single place value	$265 + 10$
Reordering	Change the order of $15 + 8 + 5$ and calculate $15 + 5 + 8$ $2 + 54$ would be better approached as $54 + 2$
Partitioning and bridging through 10	$15 + 32$ might be approached by calculating $30 + 10 + 5 + 2$ $37 + 7$ worked out by partitioning and bridging through 10 ie. $37 + 3 + 4$
Application of number facts to larger numbers	$800 + 400$ can be calculated using $8 + 4$
Partitioning and compensating (for numbers close to 10)	$38 + 69$ can be worked out using $38 + 70 - 1$
Near doubles	$38 + 35$ can be evaluated by doing $35 \times 2 + 3$

Addition using a formal written method

<table>
<tr><th colspan="2">Addition of numbers that have two or more digits using a written method</th></tr>
<tr><td>Example questions
a) $265 + 164$
b) $5.45 + 3.68$
c) $5324 + 790$</td><td>Barriers to success
Regularly assess pupils for:
• The ability to line the numbers up by place value.
• The ability to use number facts to add individual columns instead of using fingers to count on.</td></tr>
</table>

Base 10/dienes is in the early stages of understanding the written column method. It is also used to help pupils explain their understanding.

Example: $265 + 164 = 429$

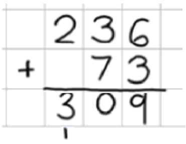


$$\begin{array}{r} 265 \\ +\,164 \\ \hline 429 \\ \hline {\scriptstyle 1} \end{array}$$

The column method is the method used when mental methods are not appropriate. Before using it, pupils should think 'can I do this calculation mentally?'

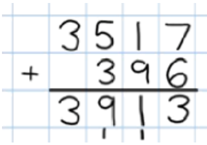
Examples:

1) $236 + 73$



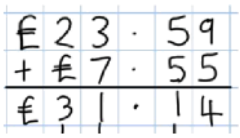
Add ones first and go from right to left. Use place value language: '3 tens add 7 tens'

2) $3517 + 396$



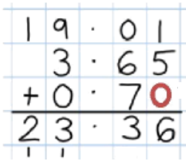
'Exchange' numbers underneath the answer

3) £23.59 + £7.55



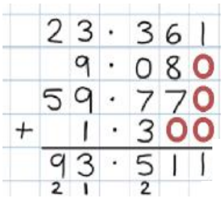
Decimal points should be aligned and included in the answer

4) $19.01 + 3.65$



Saying '6 tenths add 7 tenths' makes a clear link with place value

5) $23.361 + 9.08 + 59.77 + 1.3$



Empty decimal places can be filled with zeros to show place value

Mental Subtraction

Subtraction: mental subtraction of small numbers within 10 or 20

Example questions

a) $8 - 6$
b) $13 - 8$
c) $17 - 4$
d) **Take away** 3 from 11
e) **Subtract** 5 from 9
f) Find the **difference** between 18 and 7

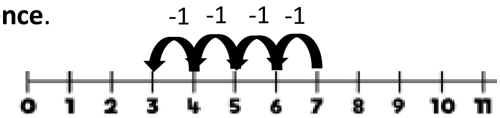
Barriers to success

Regularly assess pupils for:

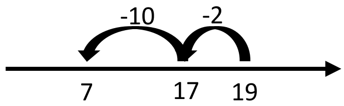
- Lack of fluency with number bonds.
- Use of number bonds to 10 to cross 10 mentally.
- A reliance on counting on fingers.
- A good knowledge of the words for subtraction.
- An understanding that the order of a subtraction is important.

Number lines show pupils the relationship between addition and subtraction. They also show subtraction as **counting back** or **difference**.

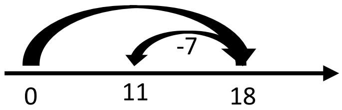
Example: $7 - 4$



Example: $19 - 12$



Example: Find the difference between 18 and 7



Subtraction: mental subtraction of larger numbers

Example questions

a) $100 - 60$
b) $84 - 19$
c) $265 - 123$
d) $83 - 79$

Barriers to success

Regularly assess pupils for:

- Lack of fluency with number bonds.
- Over reliance on written methods without first considering more efficient approaches.

Mental methods

Subtracting a single place value	$265 - 10$
Subtracting from left to right	$265 - 123$ becomes $265 - 100 - 20 - 3$
Reordering to use number facts	Changing the order of $15 - 8 - 5$ and calculating $15 - 5 - 8$
Partitioning and bridging through 10	$23 - 9$ might be solved by calculating $23 - 3 - 6$
Partitioning and compensating (for numbers close to 10)	$84 - 19$ might be approached by calculating $84 - 20 + 1$
Counting on	$83 - 79$ could be solved by counting up from 79 $400 - 135$ solved by counting on to 400 ie. $5 + 60 + 200$

Subtraction using a formal written method

Subtraction of numbers that have two or more digits using a written method

Example questions

a) 265 – 164
b) 265 – 168
c) 265 – 68
d) 1265 – 608
e) £10.00 - £7.83

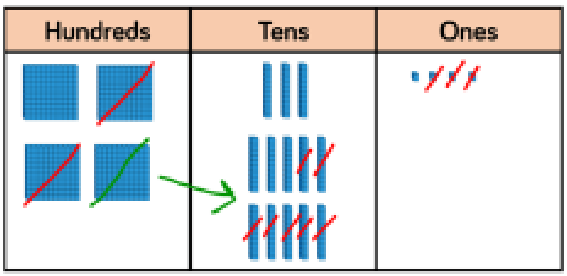
Barriers to success

Regularly assess pupils for:

- The ability to line the numbers up by place value.
- The ability to use number facts to subtract individual columns.
- A reliance on counting on fingers.
- The ability to change the order of subtraction to simplify a calculation.
- A strong understanding of how to exchange (borrow).

Base 10/dienes is in the early stages of understanding the written column method. It is also used to help pupils explain their understanding.

Example: 435 – 273




```
 3 1
  435
– 273
  262
```

The column method is the method used when mental methods are not appropriate. Before using it, pupils should think 'can I do this calculation mentally?'

Examples from the national curriculum include:

874 – 523 becomes

```
    8  7  4
 –  5  2  3
    3  5  1
```

Answer: 351

932 – 457 becomes

```
    8 12  1
    9  3  2
 –  4  5  7
    4  7  5
```

Answer: 475

932 – 457 becomes

```
    1  1
    9  3  2
 –  4  5  7
    5  6
    4  7  5
```

Answer: 475

Mental Multiplication

<table>
<tr><th colspan="2">Multiplication: mental multiplication of small numbers (between 1 and 12)</th></tr>
<tr><td>

Example questions

a) 2×3
b) 7×5
c) 9×12
d) $4\ \boldsymbol{of}\ 5$

</td><td>

Barriers to success

Regularly assess pupils for:

- Confusion between the '$\times$' and'$+$' signs.
- The use of a counting on strategy instead of applying known facts.
- The ability to change the order of a multiplication to simplify it.

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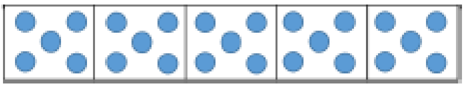
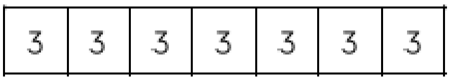
<tr><td colspan="2">

Bar models are used to show the meaning of multiplication. They make an important link with fractions and ratios. They can be used with numbers, counters or other equipment.

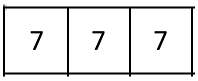
Example: 5×5



Example: 3×7



Example: 7×3



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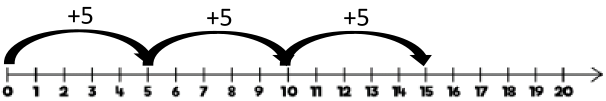
<tr><td colspan="2">

Number lines are used to show the meaning of multiplication. They make an important link with repeated addition.

Example: 3×5



</td></tr>
<tr><td colspan="2">

Arrays are used to show the meaning of multiplication. They make an important link with area. They help children to understand the commutative law ($3 \times 5 = 5 \times 3$)

Example: 3×5

</td></tr>
<tr><td colspan="2">

Memorisation of multiplication facts (tables) is essential for confidence in maths. The approach we use is:

1) Pupils are introduced to counting on in 2s, 5s and 10s
2) Pupils learn the meaning of multiplication and the multiply sign
3) Memorisation of the multiplication tables starts using Sparx. By completing weekly practice pupils will be fluent with their 1-12 tables by the end of year 4.
4) Pupils learn the relationships between facts (ie. 8 x 6 is double 4 x 6)
5) Weekly practice continues in years 5 and 6 to increase fluency and reduce the chance of forgetting.

</td></tr>
</table>

Multiplication using a formal written method

Multiplication: Multiplication of two numbers with two or more digits using a written method

Example questions

a) 23×8
b) 23×35
c) 134×27
d) What is 16 **lots of** 12
e) Work out 9 **times** 4
f) Find 45 **multiplied** by 32
g) What is the **product** of 7 and 13

Barriers to success

Regularly assess pupils for:

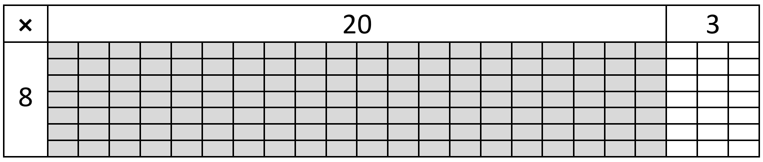
- Confusion between the '×' and'+' signs.
- A good knowledge of the words for multiplication
- A reliance on counting on strategies rather than fast recall of multiplication facts (tables)

The area model (or grid) supports the understanding of column multiplication by making partitioning explicit. It links with place value and area and is used by secondary schools to teach algebra.

Example: 23×8

×	20	3
8	160	24

$160 + 24 = 184$



Example: 136×5

×	100	30	6
5	500	150	30

$500 + 150 + 30 = 680$

Column multiplication is the most efficient method for multiplying larger numbers. It should not be taught until multiplication facts are secure.

Short multiplication is used for multiplying a large number by a single digit. It is clearly defined in the national curriculum appendices:

24 × 6 becomes

		2	4
×			6
	1	4	4
		2	

Answer: 144

342 × 7 becomes

		3	4	2
×				7
	2	3	9	4
		2	1	

Answer: 2394

2741 × 6 becomes

		2	7	4	1
×					6
	1	6	4	4	6
		4	2		

Answer: 16 446

Long multiplication is used for multiplying a large number a two or more digit number. It is clearly defined in the national curriculum appendices:

24 × 16 becomes

		2	
		2	4
×		1	6
	2	4	0
	1	4	4
	3	8	4

Answer: 384

124 × 26 becomes

		1	2	
		1	2	4
×			2	6
	2	4	8	0
		7	4	4
	3	2	2	4
	1	1		

Answer: 3224

124 × 26 becomes

		1	2	
		1	2	4
×			2	6
		7	4	4
	2	4	8	0
	3	2	2	4
	1	1		

Answer: 3224

Mental Division

Division: mental division of numbers within the 1-12 times tables

Example questions

a) $6 \div 2$
b) $35 \div 7$
c) $108 \div 12$
d) **Share** 63 into 9 **groups**
e) **Divide** 24 by 4
f) **Halve 18**

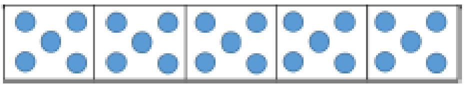
Barriers to success

Regularly assess pupils for:

- Fluent recall of the multiplication facts (tables)
- The ability to see the difference between a number of items and a number of groups
- A good knowledge of the words for division
- The ability to use multiplication facts to create division facts (eg. $3 \times 2 = 6$ so $6 \div 2 = 3$)

Bar models are used to show the meaning of division as sharing into groups. They make an important link with fractions and ratios. They can be used with numbers, counters or other equipment.

Example: $25 \div 5$



Example: $21 \div 7$

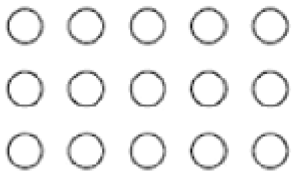
3	3	3	3	3	3	3

Example: $21 \div 3$

7	7	7

Arrays are used to show the meaning of division. They make important links with multiplication and area. They help children to see different ways of grouping objects.

Example: $15 \div 3$ and



Check pupils can count groups:
How many groups of 3?
How many groups of 5?
How many counters in each group?

Memorisation of multiplication facts (tables) is essential for confidence with division.

1) Once pupils have started learning their multiplication facts, they are taught to create and use related division facts.
2) Sparx multiplication practice include division facts at appropriate points.
3) Completing weekly practice is essential for fluency in division.

Linking understanding of division with fractions should be done systematically.
Division problems should regularly be presented in fraction form. Pupils should be taught the link between the fraction line and the division symbol.

Example: Work out $15 \div 3$ can be presented as Work out $\frac{15}{3}$

The division symbol is a fraction line with the two dots taking the position of the numerator and denominator.

$\div$ is related to $\frac{\square}{\square}$

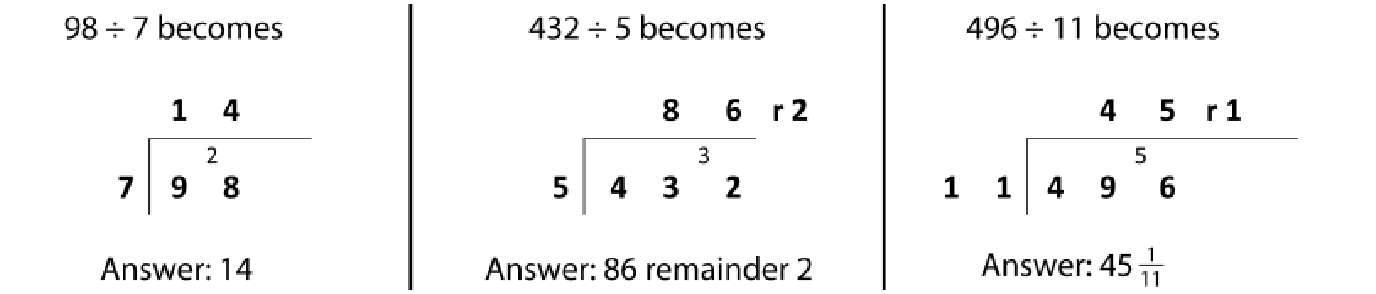
Division using a formal written method

Division: division of larger numbers using a formal written method

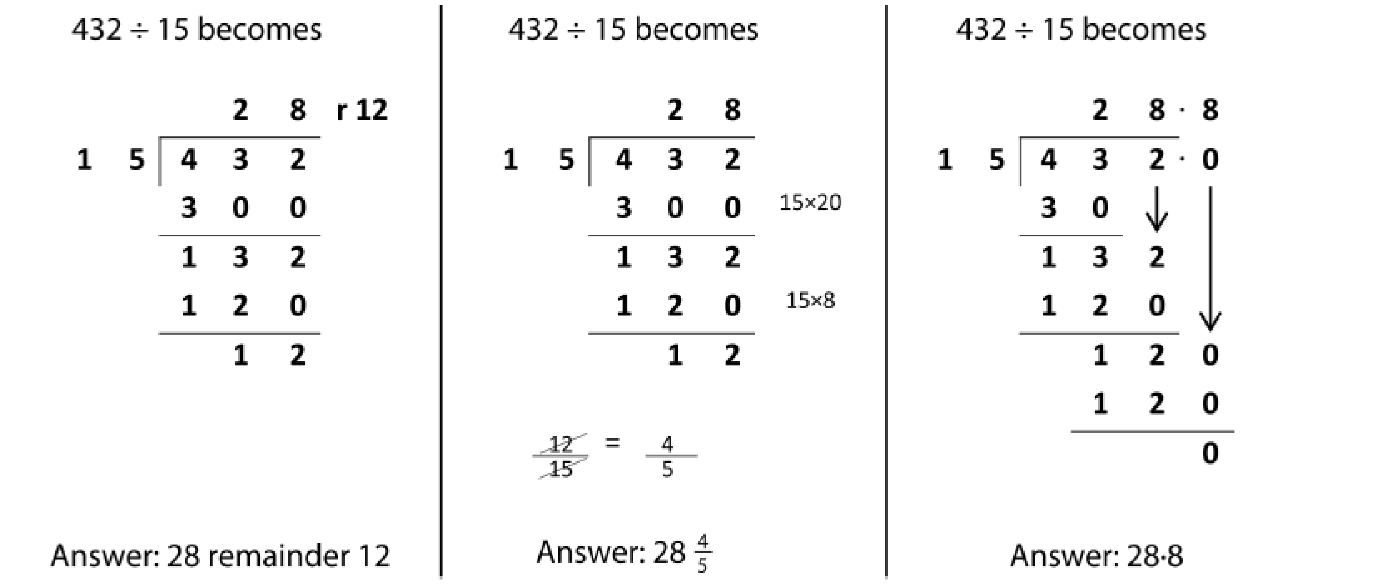
Example questions	**Barriers to success**
a) $371 \div 7$ b) $1092 \div 12$ c) **Share** 496 into 11 **groups** d) **Divide** 432 by 5	Regularly assess pupils for: • Fluent recall of the multiplication facts (tables) • The ability to see the difference between a number of items and a number of groups • The ability to use multiplication facts to create division facts (eg. $3 \times 2 = 6$ so $6 \div 2 = 3$)

Written short division is used to divide larger numbers efficiently. Before using it, pupils should think "can I do this calculation mentally?". Pupils should not be taught this unless they are fluent with the relevant multiplication facts.
Difficulty in division does not come from the length of the number. Instead it comes from the choice of number to divide by. For example, dividing by 7 will usually be more challenging than dividing by 2.

Short multiplication is used for dividing a large number by a single digit. It is clearly defined in the national curriculum appendices:



Long multiplication is used for multiplying a large number a two or more digit number. It is clearly defined in the national curriculum appendices:



Further mental multiplication and division:	
Example questions	**Barriers to success**
a) 12×10 b) 7.5×100 c) 7.05×100 d) 12×0.5 e) Double 12 f) Halve 28 g) 142×5 h) $142 \div 5$	Regularly assess pupils for: • An understanding of placeholder zeros (3.02 vs 3.20) • The misconception that multiplication always makes a number larger • The misconception that multiplying by 10 or 100 is achieved by adding zeros on to the end of a number

Multiplication and division by powers of 10. This an essential mental skill for a wide variety of future mathematical topics.

Multiplication and division by 5. Multiply by 10 and divide by 2/Divide by 10 and multiply by 2

Doubling and halving/multiplying and dividing by powers of 2. Doubling can be used to multiply by 4, 8, 16..

Use of factors to simplify multiplication
15×32 could be approached by doing $32 \times 3 \times 5$

Use of factors to simplify division
$492 \div 12$ could be approached by doing $492 \div 4 \div 3$ or $492 \div 2 \div 2 \div 3$

Understanding fractions – What is a fraction?

We want our pupils to understand that:

A fraction is a number

- It has a position on a number line
- It can be larger or smaller than one
- It can be added, subtracted, multiplied and divided (like other numbers)

A fraction can be seen as an operation

- It is something that can be 'done' to another number (what is ½ of 18)
- Finding a fraction of an amount is the same thing as multiplying that amount by the fraction.

Working with fractions of amounts

Example questions

a) $\frac{1}{2}\ of\ 18$
b) $\frac{1}{5}\ of\ 20$
c) $\frac{2}{5}\ of\ 20$
d) $\frac{12}{100}\ of\ 300$

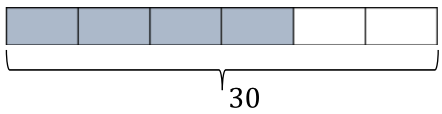
Barriers to success

Regularly assess pupils for:

- Fluent recall of the multiplication facts (tables)
- An understanding that the fraction line is the symbol for division.
- Knowing that the term '*of*' means 'multiply'.

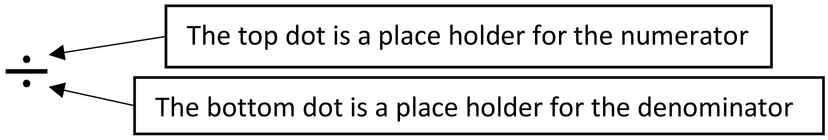
Bar models are the best way for pupils to understand fractions of an amount as they are easier to draw than the alternatives. They also support pupils with their understanding of ratio.

$\frac{4}{6}\ of\ 30$ can be represented as:



The **fraction line is the division symbol**:

$\frac{4}{6}\ means\ 4 \div 6$



Addition and subtraction with fractions

A strong understanding of **fractional equivalence is essential prior knowledge** for pupils to be able to add and subtract fractions.

- For example, Pupils should be able to explain that $\frac{4}{6}$ is exactly the same as $\frac{2}{3}$ despite being written with different numbers. They might use diagrams to explain why this is the case.
- Pupils should use fractional equivalence to make denominators larger and smaller. They should not see it as a one way process to simplify fractions.

As well as seeing it as a division, pupils should know **that the denominator of a fraction describes the 'type' of the fraction**.

Pupils should be able to explain that **it is only possible to add and subtract fractions of the same type** (denominator).

Multiplication and division with fractions

Pupils need to be taught how to choose the best way to think about multiplication in a given situation. For example, $\frac{1}{3} \times 4$ might be best understood as $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$. However, $\frac{1}{3} \times \frac{1}{2}$ would be better understood as $\frac{1}{3}\ of\ \frac{1}{2}$

Pupils need to be taught how to choose the best way to think about division in a given situation. For example, $\frac{1}{3} \div 2$ is best thought about as 'what is $\frac{1}{3}$ divided into two groups?'. Thinking of it as 'how many 2s in $\frac{1}{3}$?' is confusing. However, the reverse is true for $2 \div \frac{1}{3}$. Pupils should be taught to be aware of and choose between these representations.

Exemplar questions by year

Year 1			
Addition	**Subtraction**	**Multiplication**	**Division**
$8+2$ $11+7$ $4+6$ $9+7$ $7+9$	$10-6$ $20-8$ $6-4$ $16-9$ $16-7$	$2+2+2$	12 shared between 3
Year 2			
Addition	**Subtraction**	**Multiplication**	**Division**
$30+20$ $30+70$ $64+3$ $64+30$ $53+21$ $6+4+3$	$100-20$ $100-70$ $64-3$ $64-30$	2×3 5×4 $3\times \square = 15$	$12\div 3$ $6\div 2$
Year 3			
Addition	**Subtraction**	**Multiplication**	**Division**
$364+7$ $364+20$ $364+800$ $364+251$	$364-7$ $364-20$ $364-800$ $364-251$ $364-271$	20×4 23×8 5×10 $4\times 12\times 5$ $3\times \square = 18$	$96\div 3$ $13\div 3$ $72\div 4$
Year 4			
Addition	**Subtraction**	**Multiplication**	**Division**
$3643+2519$	$3643-2519$	$3\times 7\times 8$ 12×12 136×5 346×9 7×100 39×7	
Year 5			
Addition	**Subtraction**	**Multiplication**	**Division**
		327×8 72×38	
Year 6			
Addition	**Subtraction**	**Multiplication**	**Division**
		3.19×8 1234×16	