

Learning Now for Future Success



CPD SCHOOLS
FEDERATION

Striving for Excellence

CPD CALCULATION POLICY

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CPD Schools Federation Calculation Policy

Introduction

The purpose of this calculation policy is to provide guidance for the teaching of calculation methods in order to ensure consistency of practice in our schools and across the Federation. This policy is in accordance with The Primary National Curriculum in England. The different strategies are separated into the 4 operations of addition, subtraction, multiplication and division for ease of reference. However, it is intended that addition, subtraction, multiplication and division will be taught in other contexts to ensure that the children are making connections and seeing relationships in their mathematical skills. The strategies are with examples supporting a CPA approach, (Concrete, Pictorial, and Abstract). It is important to note that the concrete and representation examples are not exhaustive, and teachers and pupils may well come up with alternatives. For pupils to have a true understanding of a mathematical concept, they need to master all three phases (concrete-pictorial-abstract) CPA approach.

- Concrete - the doing: A pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding. Concrete refers to objects such as base ten blocks, fraction tiles, markers, or other objects that can be physically manipulated.
- Pictorial - the seeing: When a pupil has sufficiently understood the hands-on experiences performed, they can now relate them to pictorial representations, such as a diagram or picture of the problem.
- Abstract - the symbolic: A pupil is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$. This is the most formal stage of mathematical understanding. Abstract representations can simply be an efficient way of recording the maths, without being the actual maths. They need to master all three phases.

Reinforcement is achieved by going back and forth between these representations. For example, if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial. In short, these are not always 'exclusive' representations. Children should have a secure understanding before progressing onto the next step. Pupils who are successful should be challenged through rich problems rather than accelerating to the next stage unless it is deemed appropriate.

High expectations of mathematical language development is essential, promoting mathematics as a language in itself that feeds in to all elements of mathematics including reasoning. Teachers are expected to teach and understand all mathematical language related to the four operations and this is clearly set out in the Federation mathematics vocabulary list as well as on the Calculations Policy. The National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully.

EYFS Maths

The CPD federation approach to mathematics is based on the CPA method developed by American psychologist, Jerome Bruner.

Concrete

Concrete is the “doing” stage, using concrete objects to model problems.

Pictorial

Pictorial is the “seeing” stage, using representations of the objects to model problems.

Abstract

Abstract is the “symbolic” stage, where children are able to use abstract symbols to model problems.

This approach develops children’s understanding at a deeper level and helps children learn new ideas and build on their existing knowledge by introducing abstract concepts in a more familiar and tangible way.

Key Skills in Number (Nursery)

- Develop fast recognition of up to 3 objects, without having to count them individually (‘subitising’).
- Recite numbers past 5.
- Say one number for each item in order: 1,2,3,4,5.
- Know that the last number reached when counting a small set of objects tells you how many there are in total (‘cardinal principle’).
- Show ‘finger numbers’ up to 5. Link numerals and amounts: for example, showing the right number of objects to match the numeral, up to 5.
- Experiment with their own symbols and marks as well as numerals.
- Solve real world mathematical problems with numbers up to 5.
- Compare quantities using language: ‘more than’, ‘fewer than’.
- Encourage children in their own ways of recording (for example) how many balls they managed to throw through the hoop. Provide numerals nearby for reference.

Key Skills in Number (Reception)

- Count objects, actions and sounds.
- Subitise
- Show small quantities in familiar patterns (for example, dice) and random arrangements.
- Count beyond 10.
- Compare numbers.
- Understand the ‘one more than/one less than’ relationship between consecutive numbers.
- Link the number symbol (numeral) with its cardinal number value.
- Explore the composition of numbers to 10.
- Automatically recall number bonds for numbers 0–5 and some to 10.

Early Learning Goal for Mathematics:

Number ELG

Children at the expected level of development will:

- Have a deep understanding of number to 10, including the composition of each number
- Subitise (recognise quantities without counting) up to 5
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Numerical Patterns ELG

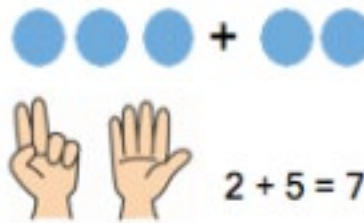
Children at the expected level of development will:

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

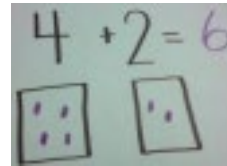
Children must be exposed to different representations of mathematical concepts in order to embed conceptual understanding. One of the aims under the Characteristics of Effective Learning is ‘creating and thinking critically.’ Children are encouraged to make links, find new ways to do things, solve problems, change strategies as needed, make predictions, develop ideas of grouping, sequencing, cause and effect.

Addition EYFS

Children begin to combine groups of objects or pictures using concrete apparatus.



Solve simple problems using fingers.



Construct number sentences verbally or using cards to go with practical activities.



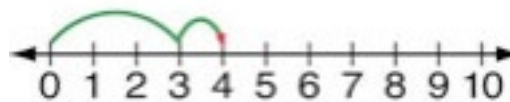
Children are encouraged to read number sentences aloud in different ways e.g. "Three add two equals 5" "5 is equal to three and two."



Count on to find the answer.

$$3 + 1 = 4$$

Have an understanding of what "more" means and be able to say what is one more than a given number.



Number tracks can be introduced to count up on and to find one more.

Children make a record in pictures, words

CPD Federation Calculation Policy: Multiplication

or symbols of addition activities.

When appropriate, Numicon shapes are introduced to identify 1 more/less, combine pieces to add and find number bonds.

Number lines can be used alongside number tracks and practical apparatus to solve addition calculations and word problems.

Vocabulary:

Games and songs can be useful way to begin using vocabulary involved in addition. Add, more, sum, and make, total, altogether.

Addition Year 1

CPA Approach
 (Concrete, Pictorial, Abstract)

Counting and Combining sets of Objects to 20
 Combining two sets of objects e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings, ten frames, etc., which will progress onto adding on to a set.

Understanding of counting using knowledge of number bonds
 Ensure that children understand the breaking up of a calculation. $2+3=5$ Represented using concrete linking to pictorial

From here move onto a bar model format so the children are used to grouping amounts together.

Understanding of counting on
 (Supported by models and images).
 $7+4$

If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown. For example a number line that starts at 20 and finishes at 30

Children should be able to separate 2 digit numbers to add the ones then add the tens.

+ = signs and missing numbers
 Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.
 $2 = 1 + 1$
 $2 + 3 = 4 + 1$
 This would be modelled well using resources to show that both sides are balanced

Vocabulary

Add, Addition ,Total, Equal, Group, Calculation, Plus, Together

Addition Year 2

CPA Approach
(Concrete, Pictorial, Abstract)

It is valuable to use a range of representations (also see Y1). Continue to use objects, number lines and ten frames to develop understanding of commutative law and of:

Counting on in tens and ones
 $23 + 12 = 23 + 10 + 2$
 $= 33 + 2$
 $= 35$

Partitioning and bridging through 10.
 The steps in addition often bridge through a multiple of 10
 E.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.
 $8 + 7 = 15$ $(8+2+5=15)$ Use Cube

Bar Modelling

Use of two parts making one whole through bar modelling. This can be shown using cubes as well as drawn.
 This will support your pictorial element

Towards a Written Method
 Partitioning in different ways and recombine to 100

Leading towards Exchanging

Standard column method:
 Concrete and pictorial to support the teaching of this method. Use of resources a necessity

Missing number problems

Whole and 2 Parts

Part + Part = Whole
 Whole - Part = Whole

| | tens | ones |
|---|------|------|
| | 2 | 3 |
| + | 1 | 4 |
| | | 7 |

$14 + 5 = 10 + \square$ $32 + \square + \square = 100$ $35 = 1 + \square + 5$

Vocabulary

Add, Addition, Total, Equal, Group, Calculation, altogether

Addition Year 3

CPA Approach
 (Concrete, Pictorial, Abstract)

Bar Modelling
 Bar Model approaches which will be continually referred to throughout the year to support the visual pictorial stage.
 Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Base 10
 Use of Base ten to support concrete and pictorial

Partition into tens and ones
 Partition both numbers and recombine.
 Count on by partitioning the second number only e.g.

Towards a Written Method to 1000
 Standard column addition can be modelled with place value counters, objects and pictorial representations

Leading to children understanding the renaming between tens and ones (carrying/exchanging).

Include the Bar Method (See Above).
 Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers

70 + 30 = 100

$20 + 11 = 31$
 $30 + 1 = 31$
 $30 + 1 = 31$

$247 + 125 = 247 + 100 + 20 + 5$
 $= 347 + 20 + 5$
 $= 367 + 5$
 $= 372$

$30 + 3$
 $+ 20 + 3$
 $50 + 6$

| | h | t | o |
|---|---|---|---|
| | | | 8 |
| + | 2 | 3 | 6 |
| | | 1 | 4 |
| | | 3 | 0 |
| + | 2 | 0 | 0 |
| | 2 | 4 | 4 |

| | h | t | o |
|---|---|---|---|
| | | 1 | |
| + | 2 | 3 | 6 |
| | 3 | 4 | 5 |
| | 5 | 8 | 1 |

Vocabulary

Addend, Total, Sum, Partition, increase, altogether

Addition Year 4

CPA Approach
 (Concrete, Pictorial, Abstract)

Mental methods (within 10,000)

This can be taught alongside methods such as Bar Modelling so that the children have a visual representation

Written methods (progressing to 4-digits & 1dp)

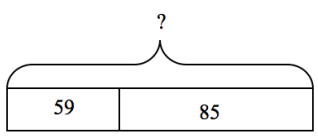
Continue to model column addition with place value counters, objects, pictorial representations and the Bar Method

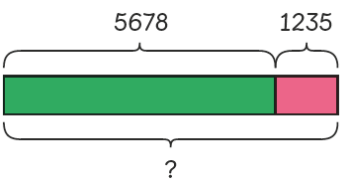
Ensure in the early stages you support as much as possible using resources to support calculations.

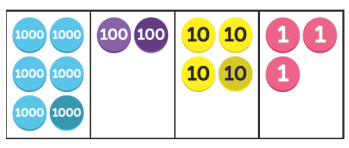
Regrouping

Extend to numbers with at least four digits, including renaming between various columns (Regrouping).

Select and use different methods to solve word problems, involving two step problems in context

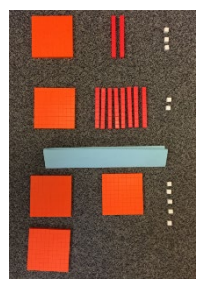






| | | | | |
|---|---|---|---|---|
| 4 | 2 | 5 | 6 | |
| + | 1 | 9 | 8 | 7 |
| | | | | |
| | | 1 | 3 | 0 |
| | | 1 | 3 | 0 |
| | | 1 | 1 | 0 |
| + | 5 | 0 | 0 | 0 |
| | | | | |
| 6 | 2 | 4 | 3 | |

1 3 add ones
 1 3 0 add tens
 1 1 0 0 add hundreds
 + 5 0 0 0 add thousands



| | | | | |
|---|---|---|---|---|
| | 1 | 1 | 1 | |
| | 4 | 2 | 5 | 6 |
| + | 1 | 9 | 8 | 7 |
| | | | | |
| | 6 | 2 | 4 | 3 |
| | | | | |

Vocabulary

Addend, Total, Sum, Partition, increase, altogether

Addition Year 5

CPA Approach
(Concrete, Pictorial, Abstract)

Mental methods (within 1,000,000)

Develop, supported by a range of models and images, including place value counters. Children should practise with increasingly large numbers to aid fluency

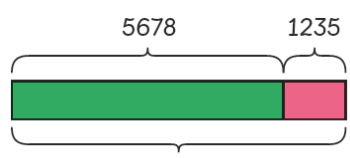
Written methods (progressing to more than 4-digits & 2dp)

As in Year 4, continue to explore column addition modelled with place value counters, objects, pictorial representations and the Bar Method

Column Additions

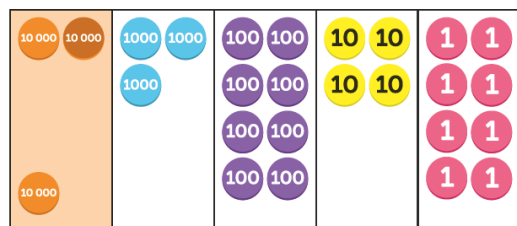
Children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written method.

Select and use different methods to solve word problems, involving two step problems in context. Use of Bar Model alongside calculations for word problems.



$5678 + 1235 = ?$

$12462 + 2500 = 14962$



| | | | | |
|---|---|---|---|---|
| | 1 | | | |
| | 1 | 6 | 6 | 0 |
| + | 1 | 7 | 2 | 4 |
| | | | | |
| | 3 | 3 | 8 | 4 |
| | | | | |
| | 3 | 3 | 8 | 4 |
| | | | | |
| | 3 | 3 | 8 | 4 |
| | | | | |
| | 3 | 3 | 8 | 4 |

Vocabulary

Addend, Total, Sum, Partition, increase, altogether

Addition Year 6

CPA Approach
 (Concrete, Pictorial, Abstract)

Mental methods

Should continue to develop, supported by a range of models and images, including the number line. Including negative numbers and decimals

Written methods

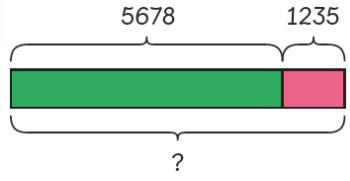
As in Year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue to model with place value counters, objects, pictorial representations and the Bar Method (See Appendix 1) Continue calculating with decimals, including those with different numbers of decimal places, and develop procedural fluency with renaming (carrying) to be secured.

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding

Refer to Year 5 for support with development groups.

Write an addition equation or a subtraction equation (your choice) to describe the diagram.



789 + 642 becomes

$$\begin{array}{r}
 789 \\
 + 642 \\
 \hline
 1431 \\
 \hline
 \end{array}$$

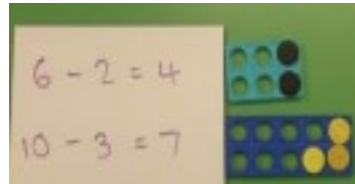
Answer: 1431

Vocabulary

Addend, Total, Sum, Partition, increase, altogether

Subtraction EYFS

Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.



Solve simple problems using fingers.

Construct number sentences verbally or using cards to go with practical activities.



Children are encouraged to read sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one."



Count back to find the answer.

Have an understanding of what "less" means and be able to say what is one less than a given number. What is 1 less than 9? 1 less than 20?

Number tracks can be introduced to count back and to find one less.

Children make a record in pictures, words or symbols of subtraction activities.

Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems.

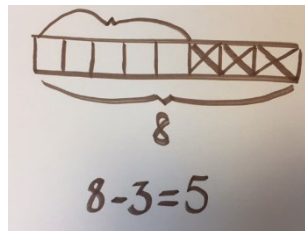
Vocabulary:

Games and songs can be a useful way to begin using the vocabulary involved in subtraction. Take (away), leave, left/left over, less, fewer, difference.

Subtraction Year 1

CPA Approach
 (Concrete, pictorial, Abstract)

Understand subtraction as crossing out (take-away) (within 20):



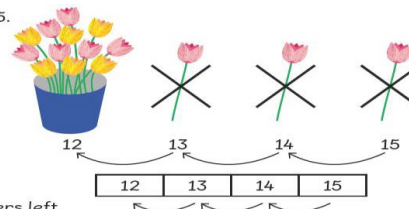
Using knowledge of number bonds to subtract (within 20):



Understand subtraction as counting back (within 20):

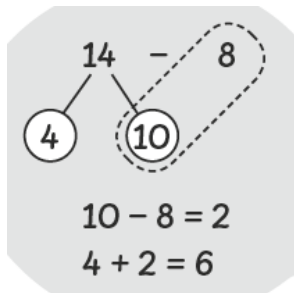
Use concrete objects and pictorial representations. Progress from using number lines with every number shown to number lines with significant numbers shown.

Subtract 3 from 15.



Partitioning to subtract

Children should be able to separate 2 digit numbers to subtract from the tens then add the leftover ones.



Missing number problems e.g.

- $7 = \square - 9$
- $20 - \square = 9$
- $15 - 9 = \square$
- $\square - \square = 11$

Ensure doing this time you are counting backwards in steps as well as counting forwards

Vocabulary

Subtraction, Subtract, Take Away, Minus, Less, Backwards,

Subtraction Year 2

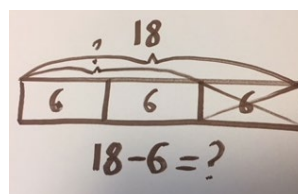
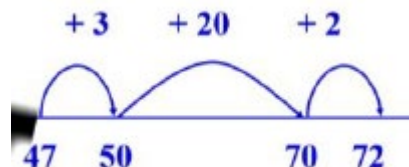
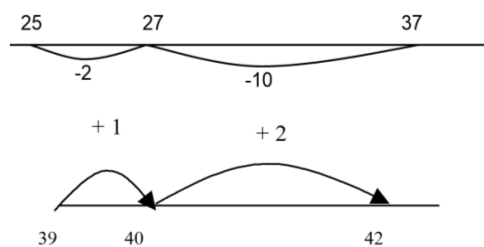
CPA Approach
(Concrete, Pictorial, Abstract)

Recall and use subtraction facts to 20 fluently

It is valuable to use a range of representations (also see Y1). Continue to use Base Ten, number lines, ten frames and objects to model take-away and difference.

The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.

Using number lines that do not need to go to 0



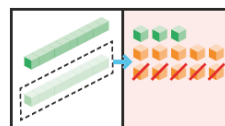
Bar Modelling by crossing out from the total

Towards written methods within 100

Record addition and subtraction in columns, the numbers may be represented with objects and pictorial representations. E.g. $23 - 5$. Progress to renaming (borrowing).

Missing number problems, including use of inverse relationships

Regroup 1 ten into 10 ones.
 Subtract the ones.
 $13 \text{ ones} - 5 \text{ ones} = 8 \text{ ones}$



| | tens | ones |
|---|--------------|--------------|
| - | 1 | 13 |
| | 1 | 3 |
| | | 5 |
| | | 8 |

$52 - 8 = \square$

$\square - 20 = 25$

$22 = \square - 21$

$6 + \square + 3 = 11$

Vocabulary

Subtraction, Subtract, Take Away, Minus, Less, Backwards, inverse

Subtraction Year 3

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods

Should continue to develop, supported by a range of models and images, including the number line
 Children should make choices about which strategy to use, depending on the numbers involved.

This will lead to renaming (borrowing), modelled using place value counters or Base Ten

Bar Model

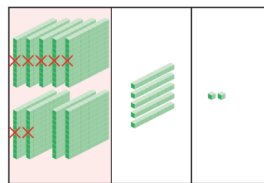
To show visualisation of subtraction problem

Written methods (progressing to 3-digits)

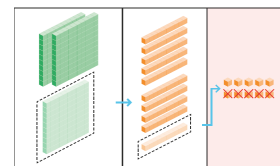
Continue to model column subtraction with no renaming (Regrouping/decomposition), modelled with objects such as place value counters, Numicon and Base ten.

3 digit - 3 digit with tens value in subtrahend being greater – exchange from hundreds (no zero place values)

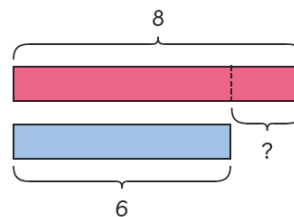
Missing number problems, including use of inverse relationships e.g.



| | h | t | o |
|---|---|---|---|
| | 9 | 7 | 5 |
| - | 7 | 2 | 3 |
| | 2 | 5 | 2 |



| | h | t | o |
|---|---|---|---|
| | 2 | 3 | 9 |
| - | 1 | 2 | 5 |
| | 1 | 2 | 4 |



$$8 - 6 = 2$$

| | h | t | o |
|---|---|---|---|
| | 9 | 7 | 5 |
| - | 7 | 2 | 3 |
| | 2 | 5 | 2 |

$$\begin{array}{r} 334 \\ - 153 \\ \hline \end{array}$$

$$\square = 43 - 27$$

$$145 - \square = 138$$

$$274 - 30 = \square$$

$$245 - \square = 195$$

$$532 - 200 = \square$$

$$364 - 153 =$$

Vocabulary

Subtraction, Subtract, Take Away, Minus, Less, Backwards, inverse
 Subtrahend (amount being taken away) Minuend (the quantity from which another will be subtracted)
 Difference.

Subtraction Year 4

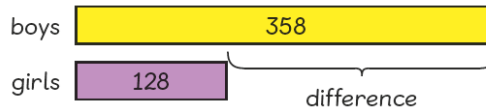
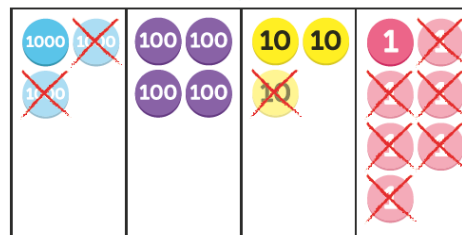
CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods (within 10,000)

Children should continue to develop, supported by a range of models and images, including partitioning.

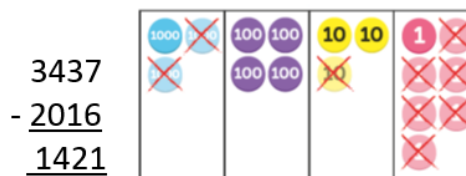
Bar Model

To support the visualisation of subtraction. The missing element as a question mark



Written methods (progressing to 4-digits & 1 dp)

Continue to use column subtraction modelled with place value counters, objects, pictorial representations and the Bar Method (See Appendix 1)



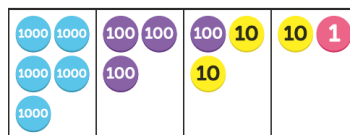
$$\begin{array}{r}
 5 \quad 2 \quad \overset{7}{\cancel{8}} \quad \overset{10}{\cancel{0}} \\
 - 3 \quad 1 \quad 6 \quad 9 \\
 \hline
 2 \quad 1 \quad 1 \quad 1
 \end{array}$$

Extend to numbers with at least four digits, including renaming between various columns (borrowing).

Use place value counters to explore compensation method

Select and use different methods to solve word problems,

Involve two step problems in context.
 Missing number/digit problems, including use of inverse relationships:



$$\begin{array}{r}
 5 \quad 3 \quad \overset{12}{\cancel{2}} \quad \overset{11}{\cancel{1}} \\
 - 1 \quad \overset{2}{\cancel{2}} \quad \overset{4}{\cancel{4}} \quad 8 \\
 \hline
 \quad \quad \quad 3 \quad 5
 \end{array}$$

$$\begin{array}{l}
 200 - 90 - 80 = \square \\
 225 - \square = 150 \\
 \square - 25 = 67 \\
 \square - 2000 = 900
 \end{array}$$

Vocabulary

Subtraction, Subtract, Take Away, Minus, Less, Backwards, inverse
 Subtrahend (amount being taken away) Minuend (the quantity from which another will be subtracted)
 Difference.

Subtraction Year 5

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods (within 1 000 000) should continue to develop, supported by a range of models and images, including partitioning.

Written methods (progressing to more than 4-digits)

As in Year 4, continue to use place value counters to support understanding of decomposition (renaming/borrowing) in formal written method. E.g. $96\ 420 - 87\ 531 =$

Continue to select and use different methods to solve word problems, involving two step problems in context.

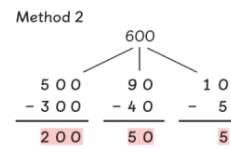
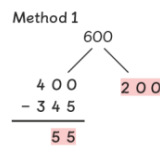
Bar Model to support problem solving

Use of bar model for missing number problems; questions with more than one answer; Numbers involving negatives.

Missing number/digit problems:

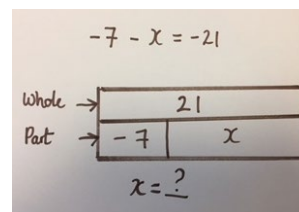
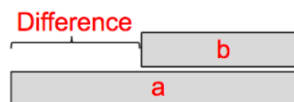
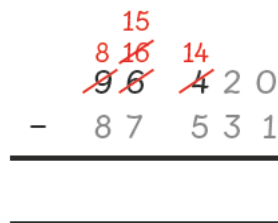
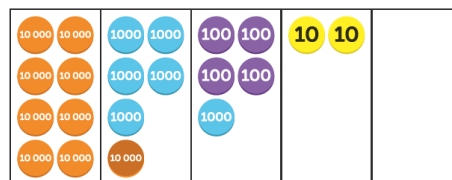
$6.45 = 6 + 0.4 + \square$;
 $119 - \square = 86$; $1\ 000\ 000 - \square = 999\ 000$;
 $600\ 000 + \square + 1000 = 671\ 000$; $12\ 462 - 2\ 300 =$

$600\ 000 - 345\ 000 =$



$600 - 345 = 200 + 55$

$600 - 345 = 200 + 50 + 5$



Vocabulary

Subtraction, Subtract, Take Away, Minus, Less, Backwards, inverse
 Subtrahend (amount being taken away) Minuend (the quantity from which another will be subtracted)
 Difference

Subtraction Year 6

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods

should continue to develop, supported by a range of models and images,

Written methods Alongside visual methods such as Bar Modelling

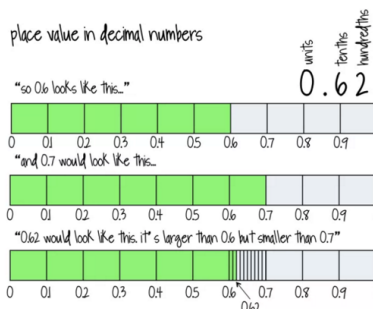
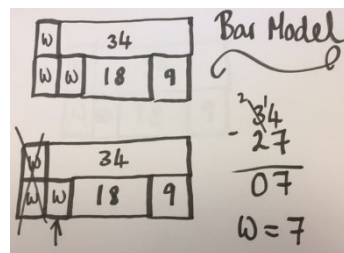
As in Year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue to model with place value counters, objects, pictorial representations and the Bar Method.

Continue calculating with decimals, including those with different numbers of decimal places, and develop procedural fluency with decomposition (Regrouping) to be secured.

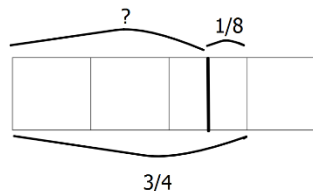
Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding

$$\begin{array}{r}
 \text{£ } 1 \overset{14}{\cancel{5}} . \overset{9}{\cancel{0}} \overset{10}{\cancel{0}} \\
 - \text{£ } 1 \quad 3 \quad . \quad 4 \quad 5 \\
 \hline
 \text{£ } \quad \square . \quad \square \quad \square
 \end{array}$$



$$3/4 \text{ of } 64 - 1/8 \text{ of } 64 =$$



Vocabulary

Subtraction, Subtract, Take Away, Minus, Less, Backwards, inverse
 Subtrahend (amount being taken away) Minuend (the quantity from which another will be subtracted)
 Difference, Zero digit
 The year group markings relate to Mental Arithmetic sessions.

The link between addition and multiplication can be introduced through doubling and reinforced through repeated addition of the same number.

Numicon is used to visualise the repeated adding of the same number.

Children begin with mostly pictorial representations e.g. How many groups of 2 are there? $2 + 2 + 2 + 2 + 2$, so 5 groups of 2.

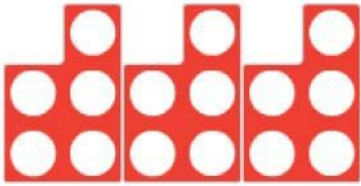


Real life contexts and use of practical equipment is used to count in repeated groups of the same size e.g. How many wheels are there altogether?

Children are encouraged to read number sentences aloud in different ways “five times two makes ten” “ten is equal to five multiplied by two” “ten is the same as five lots of two.”



Count in twos, fives and tens both aloud and with objects.

Children are given multiplication problems set in a real life context. Children are encouraged to visualise the problem e.g. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?

$5 + 5 + 5$

2, 4, 6, 8, 10, 12

Vocabulary:

Lots of, groups of, times, repeated addition, double, combine.

Multiplication Year 1

CPA Approach
 (Concrete, pictorial, Abstract)

Doubling and repeated addition

Understand multiplication is related to doubling and combining groups of the same size (repeated addition) for 2, 5, 10.

Washing line, and other practical resources for counting. Concrete objects: Base Ten, Numicon, bundles of straws, bead strings

Bar Modelling


Use of bar model as a visual to support early multiplication. This can be taught through cubes or counters

Problem solving

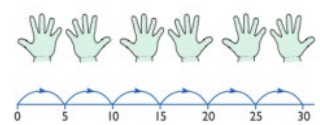
Problem solving with concrete objects (including money and measures) Use arrays to begin to understand multiplication can be done in any order (commutative)

Vocabulary

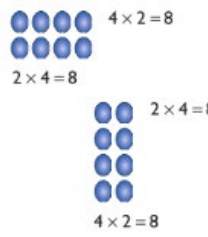
Groups of, times, multiply, addition, arrays, lots of



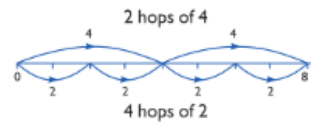
2 + 2 + 2 + 2 + 2 = 10
 $2 \times 5 = 10$
 2 multiplied by 5
 5 pairs
 5 hops of 2



$5 + 5 + 5 + 5 + 5 + 5 = 30$
 $5 \times 6 = 30$
 5 multiplied by 6
 6 groups of 5
 6 hops of 5




$4 \times 2 = 8$
 $2 \times 4 = 8$
 $2 \times 4 = 8$
 $4 \times 2 = 8$



2 hops of 4
 4 hops of 2

| | | |
|---|---|---|
| 4 | 4 | 4 |
|---|---|---|

$3 \times 4 = 12$



CPA Approach
(Concrete, Pictorial, Abstract)

Times table Facts

Recall and use multiplication facts for the 2, 5 and 10 multiplication tables as a minimum requirement. Develop understanding of solving multiplication problems using arrays, objects, pictorial representations and number lines (see Year 1).

Number Sentences

Expressing multiplication as a number sentence using \times and explore commutative law of multiplication

Scaling

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)

Number lines

Using number lines with a link to repeated addition.

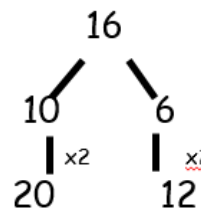
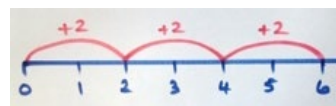
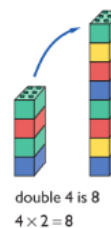
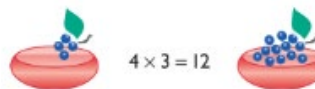
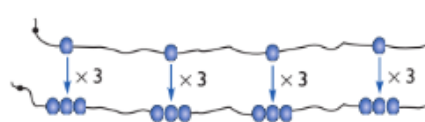
Doubling numbers up to 10 + 10

Link with understanding scaling
Use known doubles to work out double 2digit numbers
(double 15 = double 10 + double 5)

Towards written methods

Use arrays and jottings to develop an understanding of doubling two digit numbers

Solve missing number problems.



| | |
|-------------------------|-------------------------|
| $7 \times 2 = \square$ | $\square = 2 \times 7$ |
| $7 \times \square = 14$ | $14 = \square \times 7$ |
| $\square \times 2 = 14$ | $14 = 2 \times \square$ |

Vocabulary

Groups of, times, multiply, addition, arrays, lots of, multiplication

Multiplication Year 3

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods

Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts

$13 \times 4 = 10$ groups of 4 then 3 groups of 4

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables

Bar modelling

Developing this method further to go alongside calculations

Written methods (progressing to 3digit x 1digit)

Developing written methods using understanding of visual images to group and create equal groups of objects and pictures

Grid method

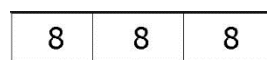
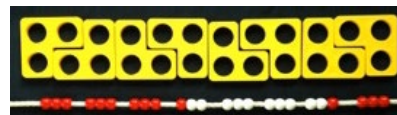
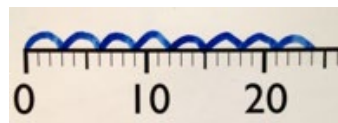
2 and 3 digit by 1 digit only looking at partitioning of multiplicand

Short Multiplication

2 by 1 digit numbers using known facts to support

Vocabulary

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by) Product, lots of, groups of, times, as much, factors



?

$3 \times 8 = ?$

| | | |
|----------|-----------|----------|
| x | 30 | 5 |
| 7 | | |

| | | | | |
|---|---|---|---|------------|
| | H | T | U | |
| | | 3 | 4 | |
| x | | | 3 | |
| | 1 | 2 | | ← (3 x 4) |
| | 9 | 0 | | ← (3 x 30) |
| = | 1 | 0 | 2 | |

24×6 becomes

| | | |
|---|---|---|
| | 2 | 4 |
| x | 6 | |
| | 1 | 4 |
| | | 4 |
| | | 2 |

Answer: 144

Multiplication Year 4

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Recall and use multiplication facts for the 6, 7, 9, 11 and 12 multiplication tables
 Use known facts to multiply by multiples of 10

Written methods (progressing to 3digit x 2digit, including 1dp)

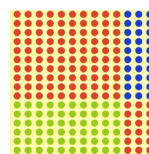
Children to embed and deepen their understanding to multiply up to 2d x 2d progressing to 3d x 2d and decimals to 1dp. Ensure this is still linked back to their understanding of arrays and supported using place value counters

Leading to short written method including regrouping (carrying):

Children to use their knowledge of multiplication tables and inverse, supported by pictorial representations and the Bar Method

Bar Model to support calculations and word problems

$$\begin{array}{r} 43 \times 6 \\ 40 \times 6 + 3 \times 6 \end{array}$$



43 x 6 by partitioning

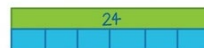
| | | |
|---|-----|----|
| x | 40 | 3 |
| 6 | 240 | 18 |

$$\begin{array}{r} 23 \\ \times \quad 6 \\ \hline 18 \\ + 120 \\ \hline 138 \end{array}$$

$$\begin{array}{r} 123 \\ \times \quad 6 \\ \hline 138 \end{array}$$

Courtney baked 24 cookies to share with her friends. If she shares her cookies with 6 friends, how many cookies would each friend get?

C = amount of cookies for each friend



Vocabulary

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by)
 Product, lots of, groups of, times, as much, factors

Multiplication Year 5

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods

X whole numbers and decimals by 10, 100, 1000 using knowledge of place value to move digits

Use practical resources and jottings to explore equivalent statements (e.g. $4 \times 35 = 2 \times 2 \times 35$)

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning)

Identify multiples and factor pairs for numbers

Written methods (progressing to 4d x 2d)

Children to continue to explore long and short methods:

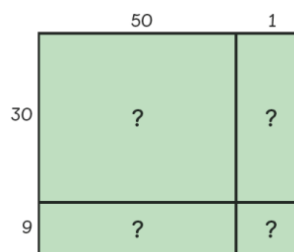
As in Year 4, children to use their knowledge of multiplication tables and inverse, supported by pictorial representations and the Bar Method

Grid method to multiply 2 digit numbers by 2 digit numbers e.g. $39 \times 51 =$

Bar model to support Problem solving. Link with other operations

$$\begin{array}{r}
 118 \\
 \times \quad 3 \\
 \hline
 24 \rightarrow \text{multiply by ones} \\
 30 \rightarrow \text{multiply by tens} \\
 + 300 \rightarrow \text{multiply by hundreds} \\
 \hline
 354
 \end{array}$$

$$\begin{array}{r}
 133 \\
 1144 \\
 \times \quad \quad 8 \\
 \hline
 9152
 \end{array}$$



Jason rode with his uncle for 6 hours at the average rate of 49 miles per hour. How far did he travel?

Jason's miles $\begin{array}{|c|c|c|c|c|c|} \hline 49 & 49 & 49 & 49 & 49 & 49 \\ \hline \end{array} \times 6 =$

$$\begin{array}{l}
 49 \times 6 = \\
 (40 \times 6) + (9 \times 6) \\
 = 240 + 54 \\
 = 294
 \end{array}$$

Jason travelled 294 miles.

Vocabulary

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by) Product, lots of, groups of, times, as much, Factors

Multiplication Year 6

CPD Federation Calculation Policy: Multiplication

CPA Approach
 (Concrete, pictorial, Abstract)

Mental methods

Identifying common factors and multiples of given numbers and prime numbers
 X 2d and 3d numbers by 1d mentally or using jottings
 Perform mental calculations including mixed operations and large numbers

Written methods

Continue to refine and deepen understanding of written methods including grid method, expanded column and fluency for using column multiplication supported by jottings and the Bar Method

Bar model to support word problems and percentages

| | | | | |
|----|-------|------|-----|----|
| X | 1000 | 300 | 40 | 2 |
| 10 | 10000 | 3000 | 400 | 20 |
| 8 | 8000 | 2400 | 320 | 16 |

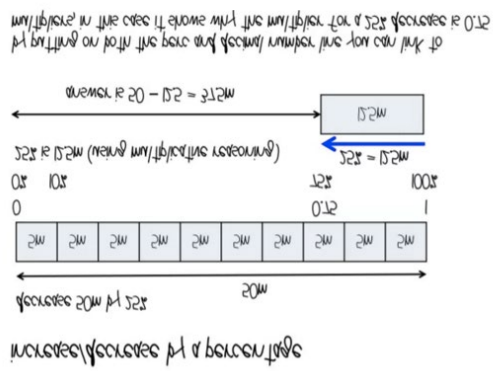
$23 \times 113 =$

$20 \times 113 = 2260$

$3 \times 113 = 339$

$23 \times 113 = 2599$

$$\begin{array}{r}
 5172 \\
 \times 38 \\
 \hline
 41376 \\
 151 \\
 + 155160 \\
 \hline
 196536 \\
 \hline
 1
 \end{array}$$



Vocabulary

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by)
 Product, lots of, groups of, times, as much, Factors

CPD Federation Calculation Policy: Multiplication

CPD Federation Calculation Policy: Multiplication

Division EYFS

Division can be introduced through halving or sharing an equal amount into 2 groups.

Children begin with mostly pictorial representations linked to real life contexts.

Children need to see and hear representations of division as both grouping and sharing.



Grouping model:

Mum has 6 socks. She grouped them into pairs. How many pairs did she make?

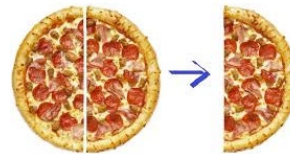


Sharing model:

I have 10 sweets. I want to share them with my friend. How many will we have each?



The sharing model is a useful way to introduce young children to fractions e.g. Can you cut the pizza in half?



Children make a record in pictures, words or symbols of division activities.

Children are encouraged to have a go at recording the calculation that has been carried out e.g. by arranging concrete objects into groups.



Vocabulary:

Half, halve, share, equal, groups of, left/left over.

Division Year 1

CPA Approach
(Concrete, pictorial, Abstract)

Number Patterns

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.
Children should be given opportunities to reason about what they notice in number patterns.

Group AND share small quantities to 10- understanding the difference between the two concepts.

Sharing

Importance of one-to-one correspondence.

Children should be taught to share using concrete apparatus.

Grouping and Use of Bar Model

Children should apply their counting skills to develop some understanding of grouping.
How many groups of 2 in 6?

Arrays as a pictorial representation can be used for division. $15 \div 3 = 5$ There are 5 groups of 3.
 $15 \div 5 = 3$ There are 3 groups of 5.

Children should be able to find $\frac{1}{2}$ of shapes, objects, numbers and quantities.

$15 \div 5 = 3$
15 shared between 5

$15 \div 2 = 7 \text{ remainder } 1$

$6 \div 2 = 3$

3 groups

Vocabulary

Division Year 2

CPA Approach
(Concrete, Pictorial, Abstract)

Sharing and Grouping

Know and understand sharing and grouping- introducing children to the \div sign.
Recall and use division facts for the 2, 5 and 10 multiplication tables

Children should continue to use grouping and sharing for division (dividends below 20) using practical apparatus, arrays and pictorial representations.

Progress to Grouping using a number line

Group from zero in jumps of the divisor to find out ‘how many groups of 3 are there in 15?’.
 $15 \div 3 = 5$

Bar Model

Using bar model to represent a number divided into $\frac{1}{2}$ $\frac{1}{4}$ etc. With not remainders at the start point.

Remainders using resources

Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?
Remainders can be introduced by use of number lines; sorting activities; cube and counter use and bar model.

1. Divide the apples into 5 bowls.

0 3 6 9 12 15

whole

part

0 1 5 9 13

Vocabulary

CPD Federation Calculation Policy: Division

Division Year 3

CPA Approach
 (Concrete, pictorial, Abstract)

Recall of Known facts

Recall and use division facts for the 3, 4 and 8 multiplication tables using times grids. Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Place value counters, arrays and number lines can be used to support children apply their knowledge of grouping.

Partitioning

Children need to be able to partition the dividend in different ways.
 $48 \div 4 = 12$

Bar Model and Short Division

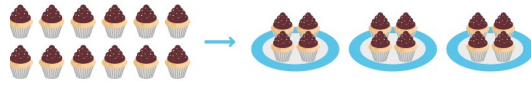
Use of bar model as a representation to link alongside basic 2 digit short division. Starting with 2 digit dividends with no remainders moving onto remainders and showing remainder as a fraction

Repeated Subtraction

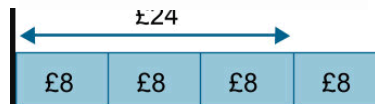
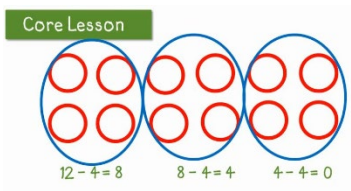
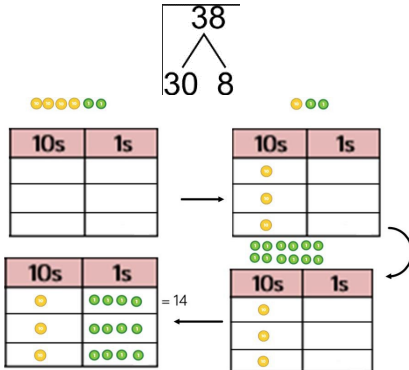
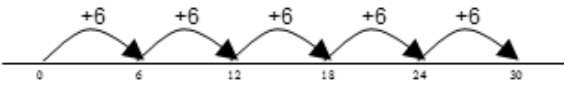
$\div =$ signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

| | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|-----|-----|-----|-----|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 | |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | |



$12 \div 4 = 3$



$3 \overline{)6393}$

Vocabulary

Divisor, dividend, quotient, sharing, groups of, lots of, vinculum

CPD Federation Calculation Policy: Division

Division Year 4

CPA Approach
 (Concrete, pictorial, Abstract)

Recall division facts for the multiplication tables up to 12 x 12

Sharing, Grouping, Repeated Subtraction and Inverse

Children will continue to explore division as sharing, grouping, repeated subtraction and inverse until they have a secure understanding. Continue to use pictorial representations and Bar Method to solve word problems in context.

Chunking method

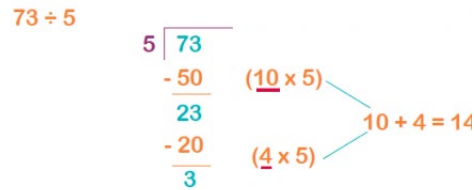
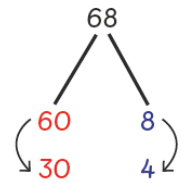
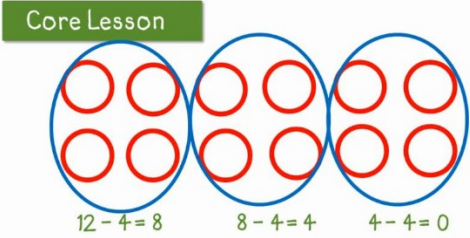
Calculations with remainders as well as without should be used

Remainders should be interpreted according to the context.

Place value and Short Division

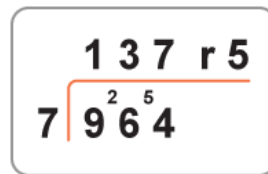
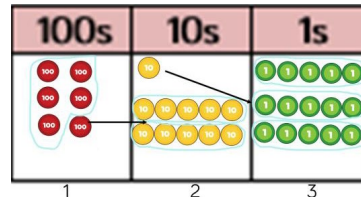
Use of partitioning and place value to show the process of regrouping across to the next column. Remainders to be represented as a remainder so:

$946 \div 7 = 137 \text{ r}5$



How many 5s have been subtracted?
 14 sets of 5, with 3 left over.

Answer: $73 \div 5 = 14 \text{ r}3$



Vocabulary

Divisor, dividend, quotient, sharing, groups of, lots of, vinculum

CPD Federation Calculation Policy: Division

Division Year 5

CPA Approach
 (Concrete, pictorial, Abstract)

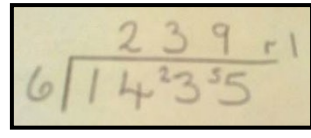
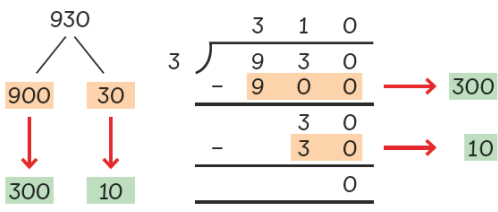
Sharing, Grouping, Repeated Subtraction and Inverse

Children should progress in their use of written division calculations:

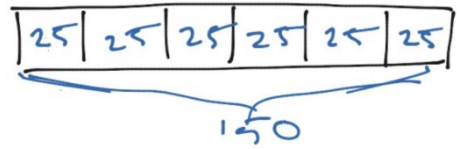
- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
 1. Dividend just over 10x the divisor, e.g. $84 \div 7$
 2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. $173 \div 15$ (learning sensible strategies for calculations such as $102 \div 17$)
 3. Dividend over 100x the divisor, e.g. $840 \div 7$
 4. Dividend over 20x the divisor, e.g. $168 \div 7$

All of the above stages should include calculations with remainders as well as without.

Children begin to practically develop their understanding of how to express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. What could I do with this remaining 1? How could I share this between 6 as well?)



$$150 \div 6 = 25$$



$432 \div 15$ becomes

$$\begin{array}{r}
 28 \text{ r}12 \\
 15 \overline{)432} \\
 \underline{30 } \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

Vocabulary

Divisor, dividend, quotient, sharing, groups of, lots of, vinculum

Division Year 6

CPA Approach
(Concrete, pictorial, Abstract)

Sharing, Grouping, Repeated Subtraction and Inverse

Children will continue to explore division as sharing, grouping, repeated subtraction and inverse and to represent problems using the Bar Method (See Appendix 1) if appropriate. Quotients (results of division) should be interpreted appropriately for the context as a whole number, remainders, decimal or fraction.

Formal Written Methods – long and short division

Continue to use partitioning, number bonds and place value counters to support the efficient use of long and short division methods including expressing remainder as a fraction or decimal
Dividing fractions by fractions

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding

$974 \div 25 =$

$$\begin{array}{r} 974 \\ 750 \\ \hline 224 \\ 200 \\ \hline 24 \end{array}$$

$$\begin{array}{r} 25 \overline{) 974} \\ \underline{- 750} \\ 224 \\ \underline{- 200} \\ 24 \end{array}$$

→ 30
→ 8

$974 \div 25 = 38 \frac{24}{25} = 38 \frac{96}{100} = 38.96$

432 ÷ 15 becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{300} \quad 15 \times 20 \\ 132 \\ \underline{120} \quad 15 \times 8 \\ 12 \end{array}$$

$\frac{12}{15} = \frac{4}{5}$

Answer: $28 \frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 28 \cdot 8 \\ 15 \overline{) 4320} \\ \underline{300} \downarrow \\ 132 \downarrow \\ 120 \downarrow \\ 120 \downarrow \\ 0 \end{array}$$

Answer: 28.8

dividing fractions by fractions

$\frac{2}{3} \div \frac{1}{6}$

$\frac{2}{3}$

"how many $\frac{1}{6}$ s go into $\frac{2}{3}$?"

$\frac{1}{6}$

= 4

Vocabulary

Divisor, dividend, quotient, sharing, groups of, lots of, vinculum